Online Exam Instruction

Here is the detailed instruction for the online exam Midterm-2.

1. It will be an open-book exam. You can only use the book, lecture videos, lecture notes and HW solutions (of your own and those we posted on course website/canvas). Any other resources/assistant are not allowed. In particular, any other resources online cannot be used and you may not collaborate or communicate (in-person or online or in any other form) with any other humans while working on this exam. Any sign of cheating/collaboration observed in the grading will be reported immediately to the Academic Integrity Office.

2. At the beginning of the exam page, you will see again this instruction and an integrity pledge. You will need to sign and date for the pledge. You don’t need to print the pledge page out. You can just copy the line “I excel with integrity” on your own paper and then sign and date it. I attach the pledge page so you can take a look in advance.

3. If you have any questions about any problem in the exam, we encourage you to use your own judgment and understanding (Of course we do our best to formulate each problem in a clean way to avoid ambiguity). If you still feel that you need to ask, then please email me (m3xiao@ucsd.edu) and cc Gongping (gniu@ucsd.edu).

4. You don’t need to print the exam out. You can just write your solutions on blank papers or latex your solutions or write on a tablet. You will need to finish and upload your solutions (handwriting scanned file, or latex generated pdf, tablet writing pdf files, or any format that is accepted by Gradescope) to Gradescope by 6:00pm on Feb. 24th.

5. When you upload your exam solution, Gradescope will ask you to match each of the problems with the page on which your solution for that problem appears. Please do it carefully and make sure they are well-matched.

6. Your submission by Gradescope is sufficient. But if for any reason you cannot upload the exam to Gradescope, then please email your exam to me (m3xiao@ucsd.edu) and cc Gongping (gniu@ucsd.edu) by 6:05pm. We will decide what to do with your exam (Do not send email if you successfully upload the exam to Gradescope). If you fail to upload via Gradescope and also fail to email your exam by 6:05pm on Feb. 24th, then your exam will not be accepted.

7. Important Tips: You are strongly recommended to leave 10 minutes (5:50pm-6:00pm) for yourself to scan and upload the exam, don’t wait until the last minute.

8. We will look for any possible signs of academic misconduct during grading the exam. If any suspicious signs are spotted, then the students may be asked to a follow-up Zoom meeting in which they will be asked to justify their work on the exam and show that it was their own work. If the follow-up is unconvincing, or the student is unable or unwilling to engage, their exam will be forwarded to the AI Office for a potential violation.
Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we’ve all wasted our time and talents!

Student Name: ____________________________________________

Excel with Integrity Pledge

I understand I may not collaborate or communicate with any other humans while working on this exam.

I am fair to my classmates and instructors by not using any unauthorized aids.

I respect myself and my university by upholding educational and evaluative goals.

I am honest in my representation of myself and of my work.

I accept responsibility for ensuring my actions are in accord with academic integrity.

I show that I am trustworthy even when no one is watching.

Affirm your adherence to this pledge by writing the following statement in the space below:

I Excel with Integrity.

Signature: ____________________________________________ Date: ____________
The exam starts here: The exam has 52 points including the bonus points, but we will grade it out of 50 points. That means we will add up all the points that you get, but the maximum total points you can have is 50.

(1) 0. Make sure you have read the exam instruction and the integrity pledge. Then copy the line “I excel with integrity” and then sign and date. You don’t need to do this on a separated paper. It can be the same papers where you are going to do the remaining questions.

(9) 1. Determine whether each of the functions is an injection, a surjection, and a bijection. Just write Y (for yes) or N (for no) in the blank. You don’t need to justify. If you write solutions in your own papers, you should write in the format like “(1) Y, Y, Y; (2). N, N, N; (3). Y, Y, Y.”

(1). \( f(x) = \sin x : [0, \frac{\pi}{2}] \to [-1, 1] \). 
Injection: ________ Surjection: ________ Bijection: ________

(2). \( f(x) = x^3 + 2 : \mathbb{R} \to \mathbb{R} \). 
Injection: ________ Surjection: ________ Bijection: ________

(3). \( f(x) = \begin{cases} -x^4 & \text{if } x \leq 0 \\ x^4 & \text{if } x \geq 0 \end{cases} \). 
Injection: ________ Surjection: ________ Bijection: ________
2. Let \( f(x) = x^2 + 1 : \mathbb{R} \to \mathbb{R} \), and \( g(x) = x^2 - 2 : \mathbb{R} \to \mathbb{R} \). Find \( f \circ g \) and \( g \circ f \). Then find \( \{x \in \mathbb{R} : f \circ g(x) = g \circ f(x)\} \). Show your work.

3. Let \( X, Y \) be two sets with \(|X| = 5, |Y| = 3\). How many surjections are there from \( X \) to \( Y \)? Show your work. You can leave your final answer as an unsimplified expression.
(16) 4 (a). Prove or disprove the following statements. You need to show your work. At the beginning of your answer, please indicate whether the statement is true or false.

(6)  (1). \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, 2x + y < 2. \)

(5)  (2). \( \forall a \in \mathbb{Z}^+, \exists b \in \mathbb{Z}^+, a^2 > b. \)

(5)  (3). \( \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x \leq (x^2 + x)y. \)
(2) \(5\) (a). Let \(T = \{n \in \mathbb{Z} : n \geq -2\}\). Find a bijection from \(T\) to \(\mathbb{Z}^+\). You need to write down an explicit formula for the bijection.

(3) \(5\) (b). Find a bijection from \((-1, +\infty)\) to \([-1, +\infty)\). You need to write down an explicit formula for the bijection.

(3) \(5\) (c). Let \(X = (0, 1) \cup (2, 3)\) and \(Y = [0, 1] \cup [2, 3]\). Prove \(X\) and \(Y\) are equipotent (Hint: Use Schröder-Bernstein theorem).
Bonus Question. The question 6 is worth only 2 point.

(1) 6 (a). Do you have any suggestions to improve the quality of future classes? Whatever suggestions you make, you will get the 1 point for this problem. You can put “No” if you do have nothing to say; and you still get 1 point. Your suggestions are highly appreciated.

(1) 6 (b). Let $2 \leq k \leq n$. Prove the following equation by using “counting in two ways”.

$$n(n-1)\binom{n-2}{k-2} = k(k-1)\binom{n}{k}.$$