Online Exam Instruction

Here is the detailed instruction for the online final exam (7:00pm-9:10pm, PT) on March 15th.

1. It will be an open-book exam. You can only use the book, lecture videos, lecture notes and HW solutions (of your own and those we posted on course website/canvas). Any other resources/assistant are not allowed. In particular, any other resources online cannot be used and you may not collaborate or communicate (in-person or online or in any other form) with any other humans while working on this exam. Any sign of cheating/collaboration observed in the grading will be reported immediately to the Academic Integrity Office.

2. On the next page, you will see an integrity pledge. You will need to sign and date for the pledge. You don’t need to print the pledge page out. You can just copy the line “I excel with integrity” on your own paper and then sign and date it. I attach the pledge page so you can take a look in advance.

3. If you have any questions about any problem in the exam, we encourage you to use your own judgment and understanding (Of course we do our best to formulate each problem in a clean way to avoid ambiguity). If you still feel that you need to ask, then please email me (m3xiao@ucsd.edu) and cc Gongping (gniu@ucsd.edu).

4. You don’t need to print the exam out. You can just write your solutions on blank papers or latex your solutions or write on a tablet. You will need to finish and upload your solutions (handwriting scanned file, or latex generated pdf, tablet writing pdf files, or any format that is accepted by Gradescope) to Gradescope by 9:10pm on March 15th, PT.

5. When you upload your exam solution, Gradescope will ask you to match each of the problems with the page on which your solution for that problem appears. Please do it carefully and make sure they are well-matched.

6. Your submission by Gradescope is sufficient. But if for any reason you cannot upload the exam to Gradescope, then please email your exam to me (m3xiao@ucsd.edu) and cc Gongping (gniu@ucsd.edu) by 9:15pm. We will decide what to do with your exam (Do not send email if you successfully upload the exam to Gradescope). If you fail to upload via Gradescope and also fail to email your exam by 9:15pm on March 15th, PT, then your exam will not be accepted.

7. Important Tips: You are strongly recommended to leave 10 minutes (or more) for yourself to scan and upload the exam, don’t wait until the last minute.

9. We will look for any possible signs of academic misconduct during grading the exam. If any suspicious signs are spotted, then the students may be asked to a follow-up Zoom meeting in which they will be asked to justify their work on the exam and show that it was their own work. If the follow-up is unconvincing, or the student is unable or unwilling to engage, their exam will be forwarded to the AI Office for a potential violation.
Excel with Integrity Pledge

The Excel with Integrity pledge affirms the UC San Diego commitment to excel with integrity both on and off campus, in academic, professional, and research endeavors.

According to the International Center for Academic Integrity, academic integrity means having the courage to act in ways that are honest, fair, responsible, respectful & trustworthy even when it is difficult. Creating work with integrity is important because otherwise we are misrepresenting our knowledge and abilities and the University is falsely certifying our accomplishments. And when this happens, the UCSD degree loses its value and we’ve all wasted our time and talents!

Student Name: ________________________________

Excel with Integrity Pledge

*I understand* I may not collaborate or communicate with any other humans while working on this exam.

*I am fair* to my classmates and instructors by not using any unauthorized aids.

*I respect* myself and my university by upholding educational and evaluative goals.

*I am honest* in my representation of myself and of my work.

*I accept responsibility* for ensuring my actions are in accord with academic integrity.

*I show that I am trustworthy* even when no one is watching.

Affirm your adherence to this pledge by writing the following statement in the space below:

*I Excel with Integrity.*

Signature: ________________________________ Date: ____________
The exam starts here: The exam has 103 points including the bonus points, but we will grade it out of 100 points. That means we will add up all the points that you get, but the maximum total points you can have is 100.

(1) 0. Make sure you have read the exam instruction and the integrity pledge. Then copy the line “I excel with integrity” and then sign and date. You don’t need to do this on a separated paper. It can be the same papers where you are going to do the remaining questions.

(4) 1. (a). State the inclusion-exclusion principle for four sets $A, B, C, D$.

(8) 1. (b). From 1 to 399, how many integers are either even or divisible by 5?
2. (a). Use Euclidean Algorithm to find the great common divisor of 120 and 168. Show your work.

2. (b). Find all solutions of the following linear congruence:

\[ 120x \equiv 48 \mod 168. \]

You are allowed to use your result in part (a). You can use any modulus as you like. Show your work.
3. Find the smallest positive integer \( k \) such that there exist \( m, n \in \mathbb{Z} \) satisfying

\[
\frac{k}{5} = 270m + 506n.
\]

Show your work.
4. (a) Find a bijection from \((-\infty, 1)\) to \([1, +\infty)\). You need to write down an explicit formula for the bijection.

4. (b) State the Schröder-Bernstein theorem.

4. (c) Let \(X = (0, 1) \cup (2, 3) \cup (4, 5)\) and \(Y = [0, 1] \cup [2, 3] \cup [5, 6]\). Prove \(X\) and \(Y\) are equipotent (You can use the above theorem in part (b)).
5. (a) Use congruence to prove the following fact:

Let $a$ be an integer. If $3$ divides $a^2$, then $3$ divides $a$.

5. (b) Use the fact in the above part (a) to prove that there is no rational number $r$ such that 

$$r^2 = 48.$$
6. (12 points) Multiple choice problem. Write your answer in the parenthesis. There is only one correct answer for every question. **You don’t need to justify.** If you write solutions in your own papers, you should write in the format like “(1) A; (2) B; (3) C; (4) D.”

(1). Let \( n = m \times 10^{1000} \), where \( m = 23, 456, 789 \). What is the remainder when \( n \) is divided by 11?

(A). 6  (B). 3  (C). 4  (D). 7

(2). Which of the following sets has the largest cardinality?

(A). \( \mathbb{Z} \);  (B). \( \mathbb{R} \);  (C). The interval \((0, 1)\);  (D). The power set of \((0, 1)\).

(3). Consider the linear congruence problem: \( 10x \equiv 24 \pmod{15} \). How many distinct solutions mod 15 it has?

(A). 0;  (B). 5;  (C). 1;  (D). infinitely many.

(4). Which of the following sets is not denumerable?

(A). The set \( \mathbb{Q} \) of rational numbers;

(B). \( \mathbb{Q} \times \mathbb{Q} \);

(C). The set of all even integers;

(D). \( \{ n \in \mathbb{Z} : n^2 < 1024 \} \).
(6) 7. (a) What is the last digit of \((2017)^{2017}\) in its decimal notation? That is, find \(0 \leq r < 10\) such that 
\[(2017)^{2017} \equiv r \mod 10.\]

(12) 7. (b) Determine whether the following statements are true or false. Write ‘True’ or ‘False’ in the parentheses. **You don’t need to justify.** If you write solutions in your own papers, you should write in the format like “(1) True; (2) False; (3) True; ⋅⋅⋅ ”

(1). Let \(a, b, c \in \mathbb{Z}^+\). Assume \(a \mid bc\), then \(a \mid b\) or \(a \mid c\). ( )

(2). The equation \(10x + 14y = 20\) has a unique integer solution. ( )

(3). Let \(a\) be an integer. If \(50 \mid a^2\), then \(50 \mid a\). ( )

(4). Let \(U = \{i \in \mathbb{Z} : 1 \leq i \leq 100\}\). There exist subsets \(A, B\) of \(U\), such that \(|A| = 52, |B| = 60\) and \(A \cap B = 10\). ( )

(5). Let \(Y\) be a finite set, and \(X\) be a proper subset of \(Y\). Then the two sets \(X\) and \(Y\) are not equipotent. ( )

(6). Let \(A\) be a set of 100 distinct integers. Then there must exist two different integers \(a, b\) in \(A\) such that 99 divides \(a - b\). ( )
(4) 8. (a) Prove that there are no integers $x$ and $y$ such that $x^4 + y^4 = 2,000,003$.

(4) 8. (b) Prove that there does not exist a positive integer $k$ such that $7k + 3$ is a perfect square. Here a perfect square means a square of some integer.
Bonus Question. The question 9 is worth only 2 points.

(2) 9. Does the following equation have any integer solution \((m, n, x, y)\)? Justify your answer.

\[30m + 42n + 70x + 105y = 1?\]