Practice Problems for Midterm 1

1. Prove the following two statements are equivalent:
   (i) \( P \Rightarrow Q \)
   (ii). \( (P \text{ or } Q) \iff Q \)

   **Hint:** Show they have the same truth table.

2. Let \( S_n = \sum_{j=1}^{n} (-1)^{j+1} j^2 \), for \( n \geq 1 \). Use induction to prove that \( S_n = (-1)^{n+1} \frac{n(n+1)}{2} \).

3. Prove the following statement:
   Let \( n \in \mathbb{Z} \). If \( n^{100} - 1 \) is even, then \( n \) is odd.

4. Prove that there do not exist integers \( m \) and \( n \) such that \( 27m + 18n = 3 \).

5. Determine whether the following statement is True or False. If true, give a proof. If false, give a counterexample.

   For any sets \( X \) and \( Y \), we have that \( \mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y) \).

6. \( \times \) Let \( A, B \) be two sets. Suppose \( A - B = B - A \). What can you say about \( A \) and \( B \)? Give a proof for your answer.

7. \( \times \) All (submitted and suggested) homework problems and examples (and exercises) in lecture notes.