Geometry of hyperbolic Cauchy-Riemann singularities and KAM-like theory for holomorphic involutions

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This work is concerned with the geometry of germs of real analytic surfaces in $(\mathbb{C}^2, 0)$ having an isolated Cauchy-Riemann singularity at the origin. We focus on perturbations of *hyperbolic* quadrics in the sense of Bishop. Contrary to the *elliptic* case, it was shown by Moser and Webster that such a surface can be transformed to a *normal form* by a change of coordinates that may not be holomorphic in any neighborhood of the origin but which is merely formal.

Given a *non-degenerate* real analytic surface M in $(\mathbb{C}^2, 0)$ having a *hyperbolic* CR singularity at the origin, we prove the existence of large amount of distinct holomorphic curves intersecting M along holomorphic hyperbolas.

This is a consequence of a non-standard KAM-like theorem for pair of germs of holomorphic involutions $\{\tau_1, \tau_2\}$. We show that such a pair has large amount of invariant analytic set biholomorphic to $\{z_1 z_2 = const\}$ (which is not a torus) in a neighborhood of the origin, and that they are conjugate to restrictions of linear maps on such invariant sets.