Koszul formula: (The existence & uniqueness of the Levi-Civita connection.)

$$\langle \nabla_X Y, Z\rangle = \frac{1}{2}\left(X\langle Y, Z\rangle + Y\langle Z, X\rangle - Z\langle X, Y\rangle\right)$$
$$+\frac{1}{2}\left(\langle [X,Y], Z\rangle - \langle [Y,Z], X\rangle + \langle [Z,X], Y\rangle\right)$$

This can be found almost in any book of Riemannian geometry (P130. Ise-Takeuchi).

Apply to our case where X, Y, Z are all left invariant on G & the inner product is Ad-invariant

$\Rightarrow$ $\langle Y, Z\rangle$ are constants, $\Longrightarrow$ First three $=0$

Hence

$$\langle \nabla_X Y, Z\rangle = \frac{1}{2}\left(\langle [X,Y], Z\rangle \underbrace{- \langle [Y,Z], X\rangle}_{\langle ad_Z Y, X\rangle} + \underbrace{\langle [Z,X], Y\rangle}_{\langle ad_Z X, Y\rangle}\right)$$

$\langle ad_Z Y, X\rangle + \langle ad_Z X, Y\rangle = 0$

$$= \frac{1}{2}\langle [X,Y], Z\rangle$$

$$\Rightarrow \quad \nabla_X Y = \frac{1}{2}[X,Y]$$

$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$$

$$= \frac{1}{2}\left(\frac{1}{2}[X,[Y,Z]] - \frac{1}{2}[Y,[X,Z]] - [[X,Y],Z]\right)$$

$-\frac{1}{2}[X,[Z,Y]]$ Jacobi

$\frac{1}{2}[Z,[Y,X]] = \frac{1}{2}[[X,Y],Z]$

$$= -\frac{1}{2}\times\frac{1}{2}\,[[X,Y],Z]$$