

Homework 2

1. Choose a domain $\Omega \subseteq [-1, 1] \times [-1, 1]$ and define the piecewise constant function $f : [-1, 1] \times [-1, 1] \rightarrow \{0, 1\}$ with $f(x) = 0$ for $x \in \Omega$ and $f(x) = 1$ otherwise. Discretize it over a grid with stepsizes $\Delta x = \Delta y = 0.01$. Now starting with an interface surrounding Ω , move a level-set function under the evolution equation

$$\phi_t = \frac{1}{1 + C|\nabla f|^2} \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right) |\nabla \phi|,$$

with large enough choice of constant C , to capture the domain.

2. Choose a piecewise continuous function $f : [-1, 1] \times [-1, 1] \rightarrow [0, 1]$ and discretize it over a grid with stepsizes $\Delta x = \Delta y = 0.01$. Add some small random noise to the discrete function at each gridpoint. Apply mean curvature flow of level-sets:

$$f_t = \nabla \cdot \left(\frac{\nabla f}{|\nabla f|} \right) |\nabla f|$$

for a small number of time steps. Check how much noise is removed from f .

3. Flow a dumbbell shape in 3D with a thin enough neck under mean curvature flow to achieve topological changes.
4. Create an initial interface that is a square in 2D and flow by unit speed in the inward normal direction ($v_n \equiv 1$):

$$\phi_t - |\nabla \phi| = 0.$$

Try to use the second-order accurate PDE solver with TVD-RK and ENO of second order.