Homework 1

1. Numerically test the order of accuracy of linear interpolation for finding the zeros, along gridlines, of $\sqrt{x^2+y^2+z^2}-0.5$ in the box $[-1,1]^3$.

2. Numerically test the order of accuracy of the formula in 2D for enclosed volume of the zero level-set:
$$\int_{[-1,1]^3} h(-\phi)\, dx$$
using
$$h(r) = \begin{cases} 0, & \text{if } x \leq -3\Delta x \\ (x+3\Delta x)/(6\Delta x), & \text{if } -3\Delta x < x < 3\Delta x \\ 1, & \text{if } x \geq 3\Delta x \end{cases}$$
for the level-set function $\phi = \sqrt{x^2+y^2}-0.5$ in the box $[-1,1]^2$ using Trapezoidal Rule for the integration.

3. (a) Make a zero level-set in the shape of a dumbbell in 2D.
   (b) Make a zero level-set in the shape of a dumbbell in 3D.

4. Consider the transport equation
$$\phi_t - \phi_x = 0 \text{ in } [-1,1]$$
with initial condition $u(x,0) = |x| - 0.5$. Observe the results using Euler's method in time and the correct 1st order upwind differencing in space when:
   (a) the time step $\Delta t =$ the spatial stepsize Δx and periodic boundary conditions are used at the boundary $x = -1, 1$.
   (b) the time step $\Delta t =$ the spatial stepsize $\Delta x/2$ and periodic boundary conditions are used at the boundary $x = -1, 1$.
   (c) the time step $\Delta t =$ the spatial stepsize Δx and von Neumann boundary conditions are used at the boundary $x = -1, 1$.
   (d) the time step $\Delta t =$ the spatial stepsize $\Delta x/2$ and von Neumann boundary conditions are used at the boundary $x = -1, 1$.

5. Consider the 2D transport equation
$$\phi_t + v \cdot \nabla\phi = 0 \text{ in } [-1,1] \times [-1,1]$$
and choose a unit vector for v. Using Euler's method in time and the correct 1st order upwind differencing in space, move a dumbbell under the velocity field v using the level-set method.