

MATH 109 – Mathematical Reasoning

Homework 6: Counting

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1. Let X and Y be finite non-empty sets, and suppose there is an injection $f: X \rightarrow Y$. Prove that $|X| \leq |Y|$.
2. Prove that a finite non-empty set of real numbers contains a minimum element.
3. Let X and Y be non-empty finite sets with $|X| = |Y|$. Prove that a function $f: X \rightarrow Y$ is injective if and only if it is surjective.
4. Let X and Y be finite sets with $|X| = n$ and $|Y| = m$. Prove that $|X \times Y| = nm$.
5. Let A and B be finite sets of real numbers with $A \subseteq B$. Prove that

$$\min B \leq \min A \leq \max A \leq \max B.$$

6. Each of a collection of 201 tiles is either triangular or square, either red or blue, and either wooden or plastic. Given that there are 101 wooden tiles, 103 red tiles, 111 triangular tiles, 50 red wooden tiles, 61 triangular wooden tiles, 57 red triangular tiles, and 36 red wooden triangular tiles, how many blue plastic square tiles are there?
7. Prove that if a set of real numbers has a maximum element, then this element is unique. [**Hint:** Suppose that c_1 and c_2 are two maximum elements of a set A and use the definition of a maximal element to prove that $c_1 = c_2$.]

Practice exercises

The following exercises are for your practice only and will not be graded. The exercises use HaLLMos, a website where you can write proofs in natural language, receive immediate feedback, and revise and resubmit as often as you like.

1. Do the following problem set on HaLLMos: Q1, Q2, Q3.
2. Use the Sandbox feature of HaLLMos to check and improve on your answers to the homework problems. This L^AT_EXguide may be helpful!