

MATH 109 – Mathematical Reasoning

Homework 1: Implications and proofs

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- Construct truth tables for the following statements.
 - $\neg(P \wedge Q)$.
 - $(\neg P) \vee (\neg Q)$.
 - $P \wedge (\neg Q)$.
 - $P \implies (P \vee Q)$.
- Using truth tables, prove that for all statements P, Q and R , the statement “ $(P \vee Q) \implies R$ ” is equivalent to the statement $(P \implies R) \wedge (Q \implies R)$.
 - Using truth tables, prove that for all statements P and Q , the statement “ $P \implies Q$ ” and its *contrapositive* “ $\neg Q \implies \neg P$ ” are equivalent.
 - What is the contrapositive of the statement “If $f(x)$ is a differentiable function, then $f(x)$ is a continuous function”?
- Let P, Q, R be statements. Write down a truth table for the statement

$$\left((P \vee Q) \implies (Q \wedge R) \right) \wedge (\neg R).$$

- Which of the following conditions are *necessary* for the positive integer n to be divisible by 6? Which are *sufficient*? Proofs are not necessary for this question.
 - 3 divides n .
 - 9 divides n .
 - 12 divides n .
 - $n = 12$.
 - 6 divides n^2 .
 - 2 divides n and 3 divides n .
 - 2 divides n or 3 divides n .
- A *tautology* is a statement which is always true. Let P and Q be statements. Which of the following four statements are tautologies? Prove that your answer is correct.
 - $\left((P \implies Q) \wedge P \right) \implies Q$.
 - $\left((P \implies Q) \wedge Q \right) \implies P$.

$$(c) \left((P \implies Q) \wedge (\neg P) \right) \implies (\neg Q).$$

$$(d) \left((P \implies Q) \wedge (\neg Q) \right) \implies (\neg P).$$

6. Let n be an integer. Prove that 0 divides n if and only if $n = 0$.
7. Let a, b, c be integers. Prove that if a divides b and a divides c , then a divides $b + c$.
8. Prove that the square of an even integer is even.