Name: ______________________________

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Math 10B: Practice Final

Print your name at the top of every page and write your PID in the space provided above.

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.
You have 3 hours.

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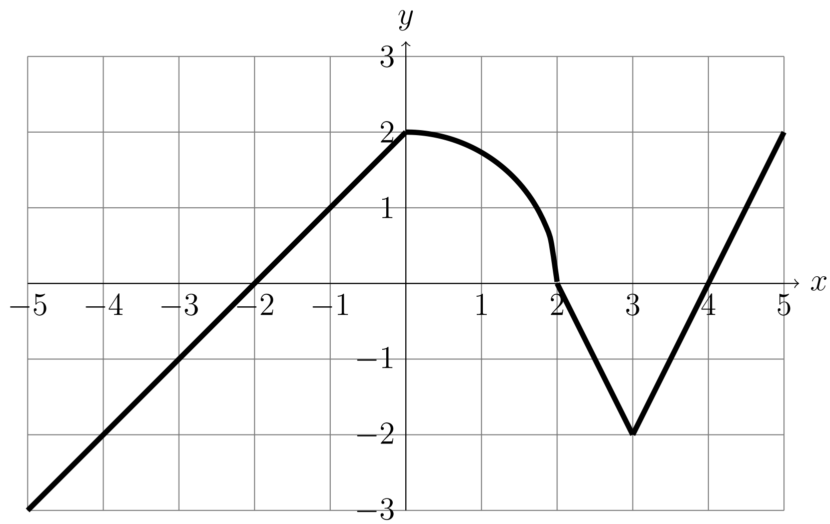
1. The graph of a function $f(x)$ is given below.



Use the graph to find $\int_{-3}^{4} f(x)\,dx$. *You may assume that curve from $x = 0$ to $x = 2$ is part of the circle $x^2 + y^2 = 4$.*

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2. Evaluate each integral.

   (a) $\displaystyle\int_{\frac{1}{5}}^{1} \frac{e^{\frac{1}{x}}}{x^2}\,dx$

   (b) $\displaystyle\int x^3\sqrt{x^2+1}\,dx$

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(c) $\displaystyle\int_0^8 |x-3|\,dx$

(d) $\displaystyle\int x^{-2}\arctan x\,dx$

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(e) $\displaystyle\int \sin^5 x \, dx$

(f) $\displaystyle\int \frac{\sqrt{x^2 - 16}}{x^4} \, dx$

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3. Let $g(x) = \int_0^x \sin t \, dt$ for $0 \leq x \leq 2\pi$.

   (a) Find $g\left(\frac{\pi}{2}\right)$.

   (b) Find $g'(x)$.

   (c) On what interval of x is $g(x)$ increasing? On what interval of x is $g(x)$ decreasing?

   (d) Find the maximum value of $g(x)$ on $[0, 2\pi]$. Find the minimum value of $g(x)$ on $[0, 2\pi]$.

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4. Determine whether each integral is convergent or divergent.

(a) $\displaystyle\int_0^\infty \frac{e^x}{1+e^{2x}}\,dx$

(b) $\displaystyle\int_1^\infty \frac{x^2}{x^3-1}\,dx$ *Hint: Use the comparison theorem.*

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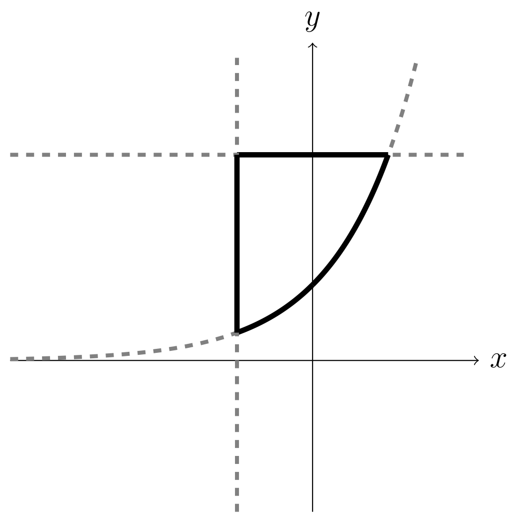
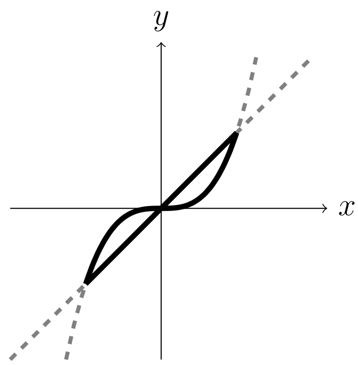
5. Find the areas of the region(s) enclosed by the given curves.

   (a) $y = e^x$, $y = e$, $x = -1$



(b) $y = x^3$, $y = x$



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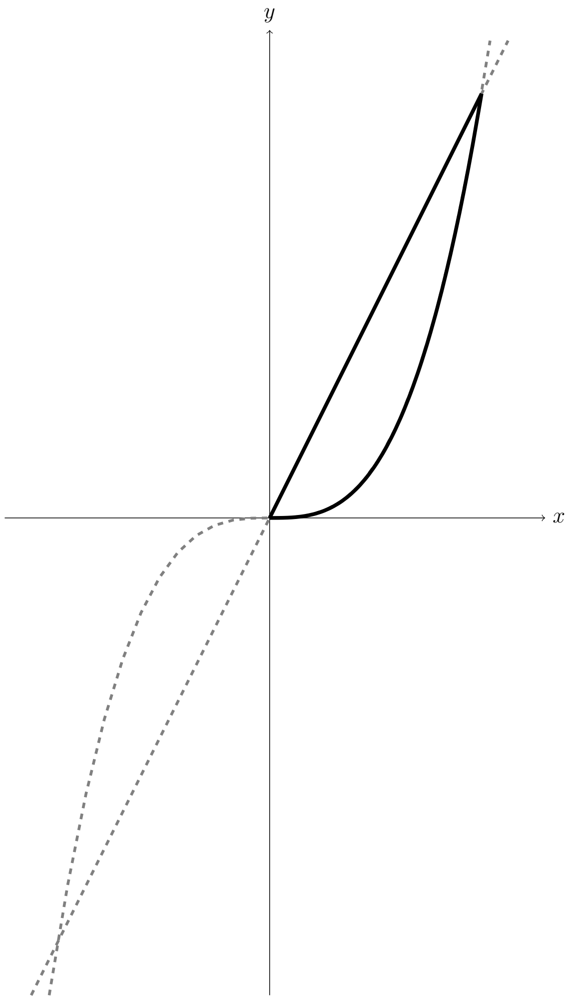
6. Set up, **but do not evaluate**, an integral for the volume of the solid of revolution obtained by revolving the region in the first quadrant bounded by $y = \frac{1}{8}x^3$ and $y = 2x$ about each of the following lines:



(a) x-axis

(b) y-axis

(c) $y = -1$

(d) $x = -1$

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7. Find the values of x for which $\sum_{n=0}^{\infty} \frac{\sin^n(2x)}{3^n}$ converges.

8. Determine whether the sequence converges or diverges. If it converges, find the limit.

(a) $a_n = \dfrac{n^5}{2-n}$

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(b) $a_n = 10 \sin\left(\frac{5}{n}\right)$

(c) $a_n = \frac{\cos(4n)}{2-n}$

(d) $a_n = \left(4^{5n+\pi}\right)^{\frac{1}{n}}$

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9. The half life of Cesium-137 is 30 years. Assume the quantity of Cesium-137 is described by the exponential model.

   (a) Find k.

   (b) If the initial quantity is 10mg, find the equation describing the mass remaining after 90 years. (Leave your answer unsimplified).

   (c) How long will it take the sample to decay to 15% of its original quantity?

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10. The half life of caffeine is approximately 6 hours. One cup of coffee contains 95 mg of caffeine. If you drink one cup of coffee at 9am, how much caffeine is still in your body at 9pm?

11. In the early part of the 20th century, seals were actively hunted under a government program that viewed them as harmful predators which greatly reduced their numbers. Subsequently, seal populations have rebounded in a roughly logistic manner. In 1975, there were 1500 seals in Washington state. The carrying capacity for this population is aroud 7500. Assume the number of seals tripled in the first year.

    (a) Assuming the size of the seal population satisfies the logistic equation, find an expression for the size of the population as a function of t where $t = 0$ corresponds to 1975.

    (b) How long will it take the population to increase to 80% of the carrying capacity?

(c) What number of seals are expected to be in Washington state after many years?

(d) Draw a direction field for this logistic equation.

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12. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+2}{n^2-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\sin\left(\frac{\pi}{4}\right)\right)^n$

13. Solve the differential equation $y' = \dfrac{y}{x}$.