**Math 103A W23 HW 9**

5. **Suggested answer**:

Q5. Prop the intersection of two normal subgroups is a normal subgroup.

pf: Suppose, $H_1, H_2 \subseteq G$ are normal subgroup

let $x \in H_1 \cap H_2$

$\therefore$ $x \in H_1$ and $x \in H_2$

$\therefore$ let $g \in G$,

$gxg^{-1} \in H_1$ and $gxg^{-1} \in H_2$

$\therefore$ $gxg^{-1} \in H_1 \cap H_2$

$\therefore$ $H_1 \cap H_2$ is a normal subgroup

8. **Suggested answer**:

Q8. To prove that $G/H$ is cyclic, we need to show that

$\exists\, gH \in G/H$, where $gH$ is the generator of $G/H$.

Let $G$ be a cyclic group. Let $H$ be some normal subgroup of $G$.

Let $g \in G$ be a generator of $G$.

Let $kH \in G/H$, $k \in G$. Since $g$ is generator of $G$, then $k = g^n$.

$(gH)^n = (gH)\dots(gH) = (g\dots g)H = (g^n)H = kH$, so $kH \in \langle gH \rangle$

Therefore $gH$ is a generator of $G/H$ so $G/H$ is cyclic.

12. **Suggested answer**:

12. G has one subgroup of order k, prove H is normal in G.

$|H| = n$

We know $gHg^{-1}$ is a subgroup of G $\forall g \in G$

And $|gHg^{-1}| = |H| = k$

So $|gHg^{-1}| = k$

And H is a normal subgroup of G.