**Math 103A W23 HW 6**

11. **Suggested answer**:

11. What are the possible cycle structures of elements in $A_5$? What about $A_6$?

The possible cycle structures of elements in $A_5$ (represented as disjoint cycles of decreasing size) are

$$(5 - cycle)$$
$$(3 - cycle)$$
$$(2 - cycle)(2 - cycle)$$
$$(id)$$

The possible cycle structures of elements in $A_6$ (represented as disjoint cycles of decreasing size) are

$$(5 - cycle)$$
$$(4 - cycle)(2 - cycle)$$
$$(3 - cycle)(3 - cycle)$$
$$(3 - cycle)$$
$$(2 - cycle)(2 - cycle)$$
$$(id)$$

37c. **Suggested answer**:

(c)

**Proposition 5.** *The order of* $r^k \in D_n$ *is* $n/\gcd(k,n)$

*Proof.* Suppose $r^k \in D_n$.
We are given that $r$ has order $n$ and thus that $\langle r \rangle$ is a cyclic group with order $n$ generated by $r$.
Therefore, Theorem 4.13 give us that the order of $r^k$ is $n/\gcd(k,n)$.
□

**common mistakes**:
Many people neglected to give justification for the differing group structures.

12. **Suggested answer**:

12.

**Proposition 8.** *If $ghg^{-1} \in H$ for all $g \in G$ and $h \in H$, then right cosets are identical to left cosets. That is, that $gH = Hg$ for all $g \in G$.*

*Proof.* Suppose $ghg^{-1} \in H$ for all $g \in G$ and $h \in H$.
Suppose $g \in G$.
Suppose $j \in gH$.
Then, we have that $j = gh$ for some $h \in H$.
Equivalently, then, we have that $j = ghg^{-1}g$ as $g^{-1}g = e$.
From above, we have that $ghg^{-1} \in H$.
Letting $h' = ghg^{-1}$, we have that $j = h'g$ and $h' \in H$.
Therefore, we have shown that $j \in Hg$ and thus that $gH \subset Hg$.
Now, suppose that $j \in Hg$.
That is, that $j = hg$ for some $h \in H$.
Then, we may saying that $j = gg^{-1}hg$ as $gg^{-1} = e$.
As $ghg^{-1} \in H$ for all $g \in G$, it follows that $g^{-1}hg = g^{-1}h(g^{-1})^{-1} \in H$ as $g^{-1} \in G$ by definition of $G$ being a group.
Thus, if we let $h' = g^{-1}hg$ we have that $j = gh'$ and $h' \in H$ and thus that $j \in gH$.
Then, we have shown that $Hg \subset gH$.
Therefore, as we have shown that $gH \subset Hg$ and $Hg \subset gH$, we have shown that $gH = Hg$. □

13. **Suggested answer**:

**Proposition 9.** *In the proof of Theorem 6.8, if $\phi : \mathcal{L}_H \to \mathcal{R}_H$ is defined by $\phi(gH) = Hg$, the proof fails because $\phi$ is not a well-defined mapping.*

*Proof.* It suffices to show there exist some subgroup $H$ of $G$ and $g_1, g_2 \in G$ such that $g_1H = g_2H$ but $Hg_1 \neq Hg_2$.
Let $G = S_3$ and $H$ be the subgroup $\{(1), (12)\}$.
Then, we may observe that $(23)H = \{(23), (132)\} = (132)H$.
However, we also may observe that $H(23) = \{(23), (123)\}$ but $H(132) = \{(132), (13)\}$ and thus $H(23) \neq H(132)$.
Therefore, the map $\phi$ is not well-defined. □