Math 103A W23 HW 3

27. **Suggested answer**:

Proposition. *The inverse of* $g_1g_2...g_n$ *is* $g_n^{-1}g_{n-1}^{-1}..g_1^{-1}$

Base case, n=1 The inverse of g_1 is g_1^{-1} by the definition of inverse.
Induction step: Assume as our IH that inverse of $g_1g_2...g_n$ is $g_n^{-1}g_{n-1}^{-1}..g_1^{-1}$ for some n greater than 1. We will show that this true for n+1.

$$g_1g_2....g_ng_{n+1} \cdot g_{n+1}^{-1}g_n^{-1}..g_1^{-1} = e$$
$$g_1g_2....g_n \cdot e \cdot g_n^{-1}g_{n-1}^{-1}..g_1^{-1} = e$$
$$g_1g_2....g_n \cdot g_n^{-1}g_{n-1}^{-1}..g_1^{-1} = e$$

And because of our Induction Hypothesis, we know that this is true. Therefore this is proved.

Comments:
Concise induction.

Common Mistakes:
Not using induction.

46. **Suggested answer**:

Proposition. *If H and K are subgroups of G, then $H \cup K$ is not necessarily a subgroup of G.*

Proof. For a proof by counterexample, consider the group G to be $\mathbb{Z}$ with the addition operation.

In this case, both $2\mathbb{Z}$ and $3\mathbb{Z}$ are subgroups of $\mathbb{Z}$.

However, since $2+3=5$ which is neither divisible by 2 or 3, the operation (addition) is not closed under the subset$(2\mathbb{Z} \cup 3\mathbb{Z})$.

This counterexample proves that $H \cup K$ is not necessarily a subgroup of G. □

Comments:
A very straightforward answer/counterexample.

54. **Suggested answer**:

(54) Proposition: Let H be the subgroup of G

$g \in G$, $gHg^{-1} = \{ghg^{-1} : h \in H\}$ is subgroup of G.

Proof:

1. Closure.

Let $x, y \in gHg^{-1}$ and $x = gh_1g^{-1}$, $y = gh_2g^{-1}$

$$(gh_1g^{-1})(gh_2g^{-1}) = gh_1(g^{-1}g)h_2g^{-1} = gh_1h_2g^{-1}$$

which is in gHg^{-1} because h_1h_2 is in H.

2. Identity

Since $e \in H$, so we have $geg^{-1} = gg^{-1} = e$.

3. Inverse

$$(ghg^{-1})(ghg^{-1})^{-1} = ghg^{-1} \cdot gh^{-1}g^{-1} = ghh^{-1}g^{-1} = gg^{-1} = e$$

Comments:
The elements of the proof are very clear: identity, inverse and closure.
Common Mistakes:
Majority of you did well! For those who didn't get full points please make sure you know how to prove a subgroup/group as this is repeatedly tested.