**Prof Wavrik** **Math 20B** **Winter 2006**

# Midterm 2 -- ANSWERS

## Part I - Short Answers

1. Give the **FORM** of the partial fractions decomposition for the following rational function, but do not evaluate the constants.

$$\frac{x^2+5}{x^2(x+1)(x^2+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1} + \frac{Dx+E}{x^2+1}$$

$$\frac{x^2+5}{x(x+1)(x^2+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{Ex+F}{x^2+1}$$

| | |
|---|---|
| Given the partial fractions decomposition<br>$\frac{x^2+2x-1}{x(x^2-1)} = \frac{1}{x} - \frac{1}{x+1} + \frac{1}{x-1}$<br>2. Find $\int \frac{x^2+2x-1}{x(x^2-1)}dx =$<br>$\ln(x) + \ln(x-1) - \ln(x+1) + C$ | Given the partial fractions decomposition<br>$\frac{x^2+x+1}{x(x^2+1)} = \frac{1}{x} + \frac{1}{x^2+1}$<br>2. Find $\int \frac{x^2+x+1}{x(x^2+1)}dx = \ln(x) + \arctan(x) + C$ |
| 3. Find $\int \frac{x^4+2x-1}{x(x^2-1)}dx =$<br>$\frac{1}{2}x^2 + \ln(x) + \ln(x-1) - \ln(x+1) + C$ | 3. Find $\int \frac{x^4+2x^2+x+1}{x(x^2+1)}dx =$<br>$\frac{1}{2}x^2 + \ln(x) + \arctan(x) + C$ |
| 4. Using trigonometric substitution x = T(t) (where T is one of sin, cos, tan, sec, csc, cotan), we can change $\int\sqrt{1-x^2}\,dx$ to one of the following integrals. Indicate which:<br>a. $\int \tan^2(t)\sec(t)\,dt$ [✓ b.] $\int \cos^2(t)\,dt$<br>c. $\int dt$ d. None of the preceding | 4. Using trigonometric substitution x = T(t) (where T is one of sin, cos, tan, sec, csc, cotan), we can change $\int\sqrt{1+x^2}\,dx$ to one of the following integrals. Indicate which:<br>[✓ a.] $\int \sec^3(t)\,dt$ b. $\int \cos^2(t)\,dt$<br>c. $\int \sec(t)\,dt$<br>d. None of the preceding |

| | |
|---|---|
| $\int \frac{x^2}{1+x^2}dx = x - \arctan(x) + C$ | $\int x\sqrt{x+1}\,dx = \frac{2}{5}(x+1)^{(5/2)} - \frac{2}{3}(x+1)^{(3/2)} + C$ |
| $\int \ln(x)\,dx = x\ln(x) - x + C$ | $\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x + C$ |
| $\int x^2 e^x\,dx = x^2 e^x - 2xe^x + 2e^x + C$ | $\int \frac{1}{x^2+x+1}dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}(2x+1)\sqrt{3}\right) + C$ |

## Part II - Evaluating Integrals