290A Final exam, Fall 2014

Three-hour exam. Answer all questions; each is worth the same. You can use standard theorems, but should say when you are doing so. Please try to write good clear mathematics; merely drawing vague pictures is not enough!

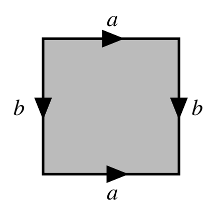
1. Let X be a topological space. Suppose γ is a path $I \to X$, and $\bar{\gamma}$ is the path with reversed orientation. Show that the composite $\gamma.\bar{\gamma}$ is homotopic, rel $\{0,1\}$, to a constant path.

2. Show that every index-3 subgroup of the free group F_2 is isomorphic to F_4.

3. Consider the inclusion $i : \mathbb{R}P^2 \to \mathbb{R}P^3$ induced by the inclusion j of S^2 as the equator in S^3. Show that i is not homotopic to a constant map.

4. How many 3-sheeted covers (not necessarily path-connected) does $S^1 \times \mathbb{R}P^3$ have, up to equivalence?

5. Let T be the torus, which we may view as a square with identification in the standard way. Let $\sigma : T \to T$ be the map induced by rotating the square 90 degrees anticlockwise about its centrepoint. Let X be the space obtained from $T \times I$ by identifying $(0, x) \sim (1, \sigma(x))$ for each $x \in T$. By finding a cell decomposition (CW complex structure) for X, compute a presentation of its fundamental group.



6. Let $L = \{(a, b) : a, b \in \mathbb{Z}\}$ be the group of integer translations acting on $\mathbb{R}^2$, and let K be its subgroup $\{(3m + n, m + 2n) : m, n \in \mathbb{Z}\}$, so that we have a covering map $\mathbb{R}^2/K \to \mathbb{R}^2/L$.

(a) How many sheets does this covering have?

(b) Letting $p \in \mathbb{R}^2/L$ be the image of $0 \in \mathbb{R}^2$, identify the fibre F at p.

(c) With respect to the canonical identification $\pi_1(\mathbb{R}^2/L, p) \cong L$, compute the monodromy homomorphism $L \to \mathrm{Aut}(F)$.