

CALCULUS III

Practice Problems

Paul Dawkins

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Preface

Here are a set of practice problems for my Calculus III notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

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Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Outline

Here is a list of sections for which problems have been written.

Three Dimensional Space

- The 3-D Coordinate System
- Equations of Lines
- Equations of Planes
- Quadric Surfaces
- Functions of Several Variables
- Vector Functions
- Calculus with Vector Functions
- Tangent, Normal and Binormal Vectors
- Arc Length with Vector Functions
- Curvature
- Velocity and Acceleration
- Cylindrical Coordinates
- Spherical Coordinates

Partial Derivatives

- Limits
- Partial Derivatives
- Interpretations of Partial Derivatives
- Higher Order Partial Derivatives
- Differentials
- Chain Rule
- Directional Derivatives

Applications of Partial Derivatives

- Tangent Planes and Linear Approximations
- Gradient Vector, Tangent Planes and Normal Lines
- Relative Minimums and Maximums
- Absolute Minimums and Maximums
- Lagrange Multipliers

Multiple Integrals

- Double Integrals
- Iterated Integrals
- Double Integrals over General Regions
- Double Integrals in Polar Coordinates
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- Triple Integrals in Cylindrical Coordinates
- Triple Integrals in Spherical Coordinates
- Change of Variables
- Surface Area
- Area and Volume Revisited

[Line Integrals](#)

[Vector Fields](#)

[Line Integrals – Part I](#)

[Line Integrals – Part II](#)

[Line Integrals of Vector Fields](#)

[Fundamental Theorem for Line Integrals](#)

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[Surface Integrals](#)

[Parametric Surfaces](#)

[Surface Integrals](#)

[Surface Integrals of Vector Fields](#)

[Stokes' Theorem](#)

[Divergence Theorem](#)

Three Dimensional Space

Introduction

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<http://tutorial.math.lamar.edu/Problems/CalcII/3DSpace.aspx>

The 3-D Coordinate System

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Equations of Lines

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Functions of Several Variables

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Calculus with Vector Functions

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Tangent, Normal and Binormal Vectors

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Arc Length with Vector Functions

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Curvature

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Velocity and Acceleration

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Cylindrical Coordinates

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Spherical Coordinates

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Partial Derivatives

Introduction

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[Limits](#)

[Partial Derivatives](#)

[Interpretations of Partial Derivatives](#)

[Higher Order Partial Derivatives](#)

[Differentials](#)

[Chain Rule](#)

[Directional Derivatives](#)

Limits

Evaluate each of the following limits.

$$1. \lim_{(x,y,z) \rightarrow (-1,0,4)} \frac{x^3 - ze^{2y}}{6x + 2y - 3z}$$

$$2. \lim_{(x,y) \rightarrow (2,1)} \frac{x^2 - 2xy}{x^2 - 4y^2}$$

$$3. \lim_{(x,y) \rightarrow (0,0)} \frac{x - 4y}{6y + 7x}$$

$$4. \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^6}{xy^3}$$

Partial Derivatives

For problems 1 – 8 find all the 1st order partial derivatives.

$$1. f(x, y, z) = 4x^3y^2 - e^z y^4 + \frac{z^3}{x^2} + 4y - x^{16}$$

$$2. w = \cos(x^2 + 2y) - e^{4x-z^4y} + y^3$$

$$3. f(u, v, p, t) = 8u^2t^3p - \sqrt{v} p^2t^{-5} + 2u^2t + 3p^4 - v$$

4. $f(u, v) = u^2 \sin(u + v^3) - \sec(4u) \tan^{-1}(2v)$

5. $f(x, z) = e^{-x} \sqrt{z^4 + x^2} - \frac{2x + 3z}{4z - 7x}$

6. $g(s, t, v) = t^2 \ln(s + 2t) - \ln(3v)(s^3 + t^2 - 4v)$

7. $R(x, y) = \frac{x^2}{y^2 + 1} - \frac{y^2}{x^2 + y}$

8. $z = \frac{p^2(r+1)}{t^3} + pr e^{2p+3r+4t}$

9. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following function.

$$x^2 \sin(y^3) + x e^{3z} - \cos(z^2) = 3y - 6z + 8$$

Interpretations of Partial Derivatives

1. Determine if $f(x, y) = x \ln(4y) + \sqrt{x+y}$ is increasing or decreasing at $(-3, 6)$ if(a) we allow x to vary and hold y fixed.(b) we allow y to vary and hold x fixed.2. Determine if $f(x, y) = x^2 \sin\left(\frac{\pi}{y}\right)$ is increasing or decreasing at $(-2, \frac{3}{4})$ if(a) we allow x to vary and hold y fixed.(b) we allow y to vary and hold x fixed.3. Write down the vector equations of the tangent lines to the traces for $f(x, y) = x e^{2x-y^2}$ at $(2, 0)$.

Higher Order Partial Derivatives

For problems 1 & 2 verify Clairaut's Theorem for the given function.

1. $f(x, y) = x^3 y^2 - \frac{4y^6}{x^3}$

2. $A(x, y) = \cos\left(\frac{x}{y}\right) - x^7 y^4 + y^{10}$

For problems 3 – 6 find all 2nd order derivatives for the given function.

3. $g(u, v) = u^3 v^4 - 2u\sqrt{v^3} + u^6 - \sin(3v)$

4. $f(s, t) = s^2 t + \ln(t^2 - s)$

5. $h(x, y) = e^{x^4 y^6} - \frac{y^3}{x}$

6. $f(x, y, z) = \frac{x^2 y^6}{z^3} - 2x^6 z + 8y^{-3} x^4 + 4z^2$

7. Given $f(x, y) = x^4 y^3 z^6$ find $\frac{\partial^6 f}{\partial y \partial z^2 \partial y \partial x^2}$.

8. Given $w = u^2 e^{-6v} + \cos(u^6 - 4u + 1)$ find w_{vuuvv} .

9. Given $G(x, y) = y^4 \sin(2x) + x^2 (y^{10} - \cos(y^2))^7$ find $G_{y y y x x x y}$.

Differentials

Compute the differential of each of the following functions.

1. $z = x^2 \sin(6y)$

2. $f(x, y, z) = \ln\left(\frac{xy^2}{z^3}\right)$

Chain Rule

1. Given the following information use the Chain Rule to determine $\frac{dz}{dt}$.

$$z = \cos(yx^2) \quad x = t^4 - 2t, \quad y = 1 - t^6$$

2. Given the following information use the Chain Rule to determine $\frac{dw}{dt}$.

$$w = \frac{x^2 - z}{y^4} \quad x = t^3 + 7, \quad y = \cos(2t), \quad z = 4t$$

3. Given the following information use the Chain Rule to determine $\frac{dz}{dx}$.

$$z = x^2y^4 - 2y \quad y = \sin(x^2)$$

4. Given the following information use the Chain Rule to determine $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

$$z = x^{-2}y^6 - 4x \quad x = u^2v, \quad y = v - 3u$$

5. Given the following information use the Chain Rule to determine z_t and z_p .

$$z = 4y \sin(2x) \quad x = 3u - p, \quad y = p^2u, \quad u = t^2 + 1$$

6. Given the following information use the Chain Rule to determine $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$.

$$w = \sqrt{x^2 + y^2} + \frac{6z}{y} \quad x = \sin(p), \quad y = p + 3t - 4s, \quad z = \frac{t^3}{s^2}, \quad p = 1 - 2t$$

7. Determine formulas for $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial v}$ for the following situation.

$$w = w(x, y) \quad x = x(p, q, s), \quad y = y(p, u, v), \quad s = s(u, v), \quad p = p(t)$$

8. Determine formulas for $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial u}$ for the following situation.

$$w = w(x, y, z) \quad x = x(t), \quad y = y(u, v, p), \quad z = z(v, p), \quad v = v(r, u), \quad p = p(t, u)$$

9. Compute $\frac{dy}{dx}$ for the following equation.

$$x^2y^4 - 3 = \sin(xy)$$

10. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following equation.

$$e^{zy} + xz^2 = 6xy^4z^3$$

11. Determine f_{uu} for the following situation.

$$f = f(x, y) \quad x = u^2 + 3v, \quad y = uv$$

Directional Derivatives

For problems 1 & 2 determine the gradient of the given function.

1. $f(x, y) = x^2 \sec(3x) - \frac{x^2}{y^3}$

2. $f(x, y, z) = x \cos(xy) + z^2 y^4 - 7xz$

For problems 3 & 4 determine $D_{\vec{u}}f$ for the given function in the indicated direction.

3. $f(x, y) = \cos\left(\frac{x}{y}\right)$ in the direction of $\vec{v} = \langle 3, -4 \rangle$

4. $f(x, y, z) = x^2 y^3 - 4xz$ in the direction of $\vec{v} = \langle -1, 2, 0 \rangle$

5. Determine $D_{\vec{u}}f(3, -1, 0)$ for $f(x, y, z) = 4x - y^2 e^{3xz}$ direction of $\vec{v} = \langle -1, 4, 2 \rangle$.

For problems 6 & 7 find the maximum rate of change of the function at the indicated point and the direction in which this maximum rate of change occurs.

6. $f(x, y) = \sqrt{x^2 + y^4}$ at $(-2, 3)$

7. $f(x, y, z) = e^{2x} \cos(y - 2z)$ at $(4, -2, 0)$

Applications of Partial Derivatives

Introduction

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[Tangent Planes and Linear Approximations](#)
[Gradient Vector, Tangent Planes and Normal Lines](#)
[Relative Minimums and Maximums](#)
[Absolute Minimums and Maximums](#)
[Lagrange Multipliers](#)

Tangent Planes and Linear Approximations

1. Find the equation of the tangent plane to $z = x^2 \cos(\pi y) - \frac{6}{xy^2}$ at $(2, -1)$.
2. Find the equation of the tangent plane to $z = x\sqrt{x^2 + y^2} + y^3$ at $(-4, 3)$.
3. Find the linear approximation to $z = 4x^2 - ye^{2x+y}$ at $(-2, 4)$.

Gradient Vector, Tangent Planes and Normal Lines

1. Find the tangent plane and normal line to $x^2y = 4ze^{x+y} - 35$ at $(3, -3, 2)$.
2. Find the tangent plane and normal line to $\ln\left(\frac{x}{2y}\right) = z^2(x - 2y) + 3z + 3$ at $(4, 2, -1)$.

Relative Minimums and Maximums

Find and classify all the critical points of the following functions.

1. $f(x, y) = (y - 2)x^2 - y^2$
2. $f(x, y) = 7x - 8y + 2xy - x^2 + y^3$
3. $f(x, y) = (3x + 4x^3)(y^2 + 2y)$
4. $f(x, y) = 3y^3 - x^2y^2 + 8y^2 + 4x^2 - 20y$

Absolute Minimums and Maximums

1. Find the absolute minimum and absolute maximum of $f(x, y) = 192x^3 + y^2 - 4xy^2$ on the triangle with vertices $(0, 0)$, $(4, 2)$ and $(-2, 2)$.
2. Find the absolute minimum and absolute maximum of $f(x, y) = (9x^2 - 1)(1 + 4y)$ on the rectangle given by $-2 \leq x \leq 3$, $-1 \leq y \leq 4$.

Lagrange Multipliers

1. Find the maximum and minimum values of $f(x, y) = 81x^2 + y^2$ subject to the constraint $4x^2 + y^2 = 9$.
2. Find the maximum and minimum values of $f(x, y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.
3. Find the maximum and minimum values of $f(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.
4. Find the maximum and minimum values of $f(x, y, z) = xyz$ subject to the constraint $x + 9y^2 + z^2 = 4$. Assume that $x \geq 0$ for this problem. Why is this assumption needed?

5. Find the maximum and minimum values of $f(x, y, z) = 3x^2 + y$ subject to the constraints $4x - 3y = 9$ and $x^2 + z^2 = 9$.

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[Surface Area](#)

[Area and Volume Revisited](#)

Double Integrals

1. Use the Midpoint Rule to estimate the volume under $f(x, y) = x^2 + y$ and above the rectangle given by $-1 \leq x \leq 3$, $0 \leq y \leq 4$ in the xy -plane. Use 4 subdivisions in the x direction and 2 subdivisions in the y direction.

Iterated Integrals

1. Compute the following double integral over the indicated rectangle **(a)** by integrating with respect to x first and **(b)** by integrating with respect to y first.

$$\iint_R 12x - 18y \, dA \quad R = [-1, 4] \times [2, 3]$$

For problems 2 – 8 compute the given double integral over the indicated rectangle.

$$2. \iint_R 6y\sqrt{x} - 2y^3 \, dA \quad R = [1, 4] \times [0, 3]$$

$$3. \iint_R \frac{e^x}{2y} - \frac{4x-1}{y^2} \, dA \quad R = [-1, 0] \times [1, 2]$$

$$4. \iint_R \sin(2x) - \frac{1}{1+6y} \, dA \quad R = \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \times [0, 1]$$

$$5. \iint_R ye^{y^2-4x} \, dA \quad R = [0, 2] \times [0, \sqrt{8}]$$

$$6. \iint_R xy^2 \sqrt{x^2 + y^3} \, dA \quad R = [0, 3] \times [0, 2]$$

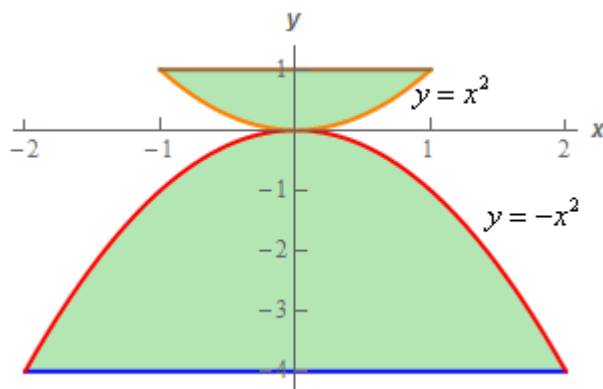
$$7. \iint_R xy \cos(yx^2) \, dA \quad R = [-2, 3] \times [-1, 1]$$

$$8. \iint_R xy \cos(y) - x^2 \, dA \quad R = [1, 2] \times \left[\frac{\pi}{2}, \pi\right]$$

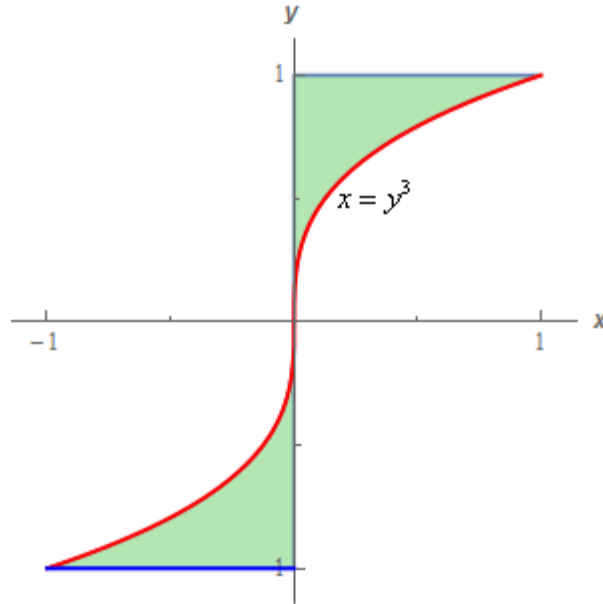
9. Determine the volume that lies under $f(x, y) = 9x^2 + 4xy + 4$ and above the rectangle given by $[-1, 1] \times [0, 2]$ in the xy -plane.

Double Integrals Over General Regions

1. Evaluate $\iint_D 42y^2 - 12x \, dA$ where $D = \{(x, y) \mid 0 \leq x \leq 4, (x-2)^2 \leq y \leq 6\}$
2. Evaluate $\iint_D 2yx^2 + 9y^3 \, dA$ where D is the region bounded by $y = \frac{2}{3}x$ and $y = 2\sqrt{x}$.
3. Evaluate $\iint_D 10x^2y^3 - 6 \, dA$ where D is the region bounded by $x = -2y^2$ and $x = y^3$.
4. Evaluate $\iint_D x(y-1) \, dA$ where D is the region bounded by $y = 1 - x^2$ and $y = x^2 - 3$.
5. Evaluate $\iint_D 5x^3 \cos(y^3) \, dA$ where D is the region bounded by $y = 2$, $y = \frac{1}{4}x^2$ and the y -axis.
6. Evaluate $\iint_D \frac{1}{y^{\frac{1}{3}}(x^3+1)} \, dA$ where D is the region bounded by $x = -y^{\frac{1}{3}}$, $x = 3$ and the x -axis.
7. Evaluate $\iint_D 3 - 6xy \, dA$ where D is the region shown below.



8. Evaluate $\iint_D e^{y^4} \, dA$ where D is the region shown below.



9. Evaluate $\iint_D 7x^2 + 14y \, dA$ where D is the region bounded by $x = 2y^2$ and $x = 8$ in the order given below.
- Integrate with respect to x first and then y .
 - Integrate with respect to y first and then x .

For problems 10 & 11 evaluate the given integral by first reversing the order of integration.

10. $\int_0^3 \int_{2x}^6 \sqrt{y^2 + 2} \, dy \, dx$

11. $\int_0^1 \int_{-\sqrt{y}}^{y^2} 6x - y \, dx \, dy$

12. Use a double integral to determine the area of the region bounded by $y = 1 - x^2$ and $y = x^2 - 3$.
13. Use a double integral to determine the volume of the region that is between the xy -plane and $f(x, y) = 2 + \cos(x^2)$ and is above the triangle with vertices $(0, 0)$, $(6, 0)$ and $(6, 2)$.
14. Use a double integral to determine the volume of the region bounded by $z = 6 - 5x^2$ and the planes $y = 2x$, $y = 2$, $x = 0$ and the xy -plane.
15. Use a double integral to determine the volume of the region formed by the intersection of the two cylinders $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$.

Double Integrals in Polar Coordinates

1. Evaluate $\iint_D y^2 + 3x \, dA$ where D is the region in the 3rd quadrant between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

2. Evaluate $\iint_D \sqrt{1 + 4x^2 + 4y^2} \, dA$ where D is the bottom half of $x^2 + y^2 = 16$.

3. Evaluate $\iint_D 4xy - 7 \, dA$ where D is the portion of $x^2 + y^2 = 2$ in the 1st quadrant.

4. Use a double integral to determine the area of the region that is inside $r = 4 + 2 \sin \theta$ and outside $r = 3 - \sin \theta$.

5. Evaluate the following integral by first converting to an integral in polar coordinates.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^0 e^{x^2+y^2} \, dy \, dx$$

6. Use a double integral to determine the volume of the solid that is inside the cylinder $x^2 + y^2 = 16$, below $z = 2x^2 + 2y^2$ and above the xy -plane.

7. Use a double integral to determine the volume of the solid that is bounded by $z = 8 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 4$.

Triple Integrals

1. Evaluate $\int_2^3 \int_{-1}^4 \int_1^0 4x^2y - z^3 \, dz \, dy \, dx$

2. Evaluate $\int_0^1 \int_0^{z^2} \int_0^3 y \cos(z^5) \, dx \, dy \, dz$

3. Evaluate $\iiint_E 6z^2 \, dV$ where E is the region below $4x + y + 2z = 10$ in the first octant.

4. Evaluate $\iiint_E 3 - 4x \, dV$ where E is the region below $z = 4 - xy$ and above the region in the xy -plane defined by $0 \leq x \leq 2$, $0 \leq y \leq 1$.

5. Evaluate $\iiint_E 12y - 8x \, dV$ where E is the region behind $y = 10 - 2z$ and in front of the region in the xz -plane bounded by $z = 2x$, $z = 5$ and $x = 0$.

6. Evaluate $\iiint_E yz \, dV$ where E is the region bounded by $x = 2y^2 + 2z^2 - 5$ and the plane $x = 1$.

7. Evaluate $\iiint_E 15z \, dV$ where E is the region between $2x + y + z = 4$ and $4x + 4y + 2z = 20$ that is in front of the region in the yz -plane bounded by $z = 2y^2$ and $z = \sqrt{4y}$.

8. Use a triple integral to determine the volume of the region below $z = 4 - xy$ and above the region in the xy -plane defined by $0 \leq x \leq 2$, $0 \leq y \leq 1$.

9. Use a triple integral to determine the volume of the region that is below $z = 8 - x^2 - y^2$ above $z = -\sqrt{4x^2 + 4y^2}$ and inside $x^2 + y^2 = 4$.

Triple Integrals in Cylindrical Coordinates

1. Evaluate $\iiint_E 4xy \, dV$ where E is the region bounded by $z = 2x^2 + 2y^2 - 7$ and $z = 1$.

2. Evaluate $\iiint_E e^{-x^2-z^2} \, dV$ where E is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 9$ with $1 \leq y \leq 5$ and $z \leq 0$.

3. Evaluate $\iiint_E z \, dV$ where E is the region between the two planes $x + y + z = 2$ and $x = 0$ and inside the cylinder $y^2 + z^2 = 1$.

4. Use a triple integral to determine the volume of the region below $z = 6 - x$, above $z = -\sqrt{4x^2 + 4y^2}$ inside the cylinder $x^2 + y^2 = 3$ with $x \leq 0$.

5. Evaluate the following integral by first converting to an integral in cylindrical coordinates.

$$\int_0^{\sqrt{5}} \int_{-\sqrt{5-x^2}}^0 \int_{x^2+y^2-11}^{9-3x^2-3y^2} 2x-3y \, dz \, dy \, dx$$

Triple Integrals in Spherical Coordinates

1. Evaluate $\iiint_E 10xz + 3 \, dV$ where E is the region portion of $x^2 + y^2 + z^2 = 16$ with $z \geq 0$.
2. Evaluate $\iiint_E x^2 + y^2 \, dV$ where E is the region portion of $x^2 + y^2 + z^2 = 4$ with $y \geq 0$.
3. Evaluate $\iiint_E 3z \, dV$ where E is the region below $x^2 + y^2 + z^2 = 1$ and inside $z = \sqrt{x^2 + y^2}$.
4. Evaluate $\iiint_E x^2 \, dV$ where E is the region above $x^2 + y^2 + z^2 = 36$ and inside $z = -\sqrt{3x^2 + 3y^2}$.
5. Evaluate the following integral by first converting to an integral in cylindrical coordinates.

$$\int_{-1}^0 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{6x^2+6y^2}}^{\sqrt{7-x^2-y^2}} 18y \, dz \, dy \, dx$$

Change of Variables

For problems 1 – 3 compute the Jacobian of each transformation.

1. $x = 4u - 3v^2$ $y = u^2 - 6v$

2. $x = u^2v^3$ $y = 4 - 2\sqrt{u}$

3. $x = \frac{v}{u}$ $y = u^2 - 4v^2$

4. If R is the region inside $\frac{x^2}{4} + \frac{y^2}{36} = 1$ determine the region we would get applying the transformation $x = 2u$, $y = 6v$ to R .

5. If R is the parallelogram with vertices $(1,0)$, $(4,3)$, $(1,6)$ and $(-2,3)$ determine the region we would get applying the transformation $x = \frac{1}{2}(v-u)$, $y = \frac{1}{2}(v+u)$ to R .

6. If R is the region bounded by $xy = 1$, $xy = 3$, $y = 2$ and $y = 6$ determine the region we would get applying the transformation $x = \frac{v}{6u}$, $y = 2u$ to R .
7. Evaluate $\iint_R xy^3 dA$ where R is the region bounded by $xy = 1$, $xy = 3$, $y = 2$ and $y = 6$ using the transformation $x = \frac{v}{6u}$, $y = 2u$.
8. Evaluate $\iint_R 6x - 3y dA$ where R is the parallelogram with vertices $(2,0)$, $(5,3)$, $(6,7)$ and $(3,4)$ using the transformation $x = \frac{1}{3}(v-u)$, $y = \frac{1}{3}(4v-u)$ to R .
9. Evaluate $\iint_R x + 2y dA$ where R is the triangle with vertices $(0,3)$, $(4,1)$ and $(2,6)$ using the transformation $x = \frac{1}{2}(u-v)$, $y = \frac{1}{4}(3u+v+12)$ to R .
10. Derive the transformation used in problem 8.
11. Derive a transformation that will convert the triangle with vertices $(1,0)$, $(6,0)$ and $(3,8)$ into a right triangle with the right angle occurring at the origin of the uv system.

Surface Area

- Determine the surface area of the portion of $2x + 3y + 6z = 9$ that is in the 1st octant.
- Determine the surface area of the portion of $z = 13 - 4x^2 - 4y^2$ that is above $z = 1$ with $x \leq 0$ and $y \leq 0$.
- Determine the surface area of the portion of $z = 3 + 2y + \frac{1}{4}x^4$ that is above the region in the xy -plane bounded by $y = x^5$, $x = 1$ and the y -axis.
- Determine the surface area of the portion of $y = 2x^2 + 2z^2 - 7$ that is inside the cylinder $x^2 + z^2 = 4$.
- Determine the surface area region formed by the intersection of the two cylinders $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$.

Area and Volume Revisited

The intent of the section was just to “recap” the various area and volume formulas from this chapter and so no problems have been (or likely will be in the near future) written.

Line Integrals

Introduction

Here are a set of practice problems for the Line Integrals chapter of my Calculus III notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

13. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,
14. Go to the download page for the site <http://tutorial.math.lamar.edu/download.aspx> and select the section you'd like solutions for and a link will be provided there.
15. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select “Printable View” from the “Solution Pane Options” to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have practice problems written for them.

[Vector Fields](#)

[Line Integrals – Part I](#)

[Line Integrals – Part II](#)

[Line Integrals of Vector Fields](#)

[Fundamental Theorem for Line Integrals](#)

[Conservative Vector Fields](#)

[Green's Theorem](#)

[Curl and Divergence](#)

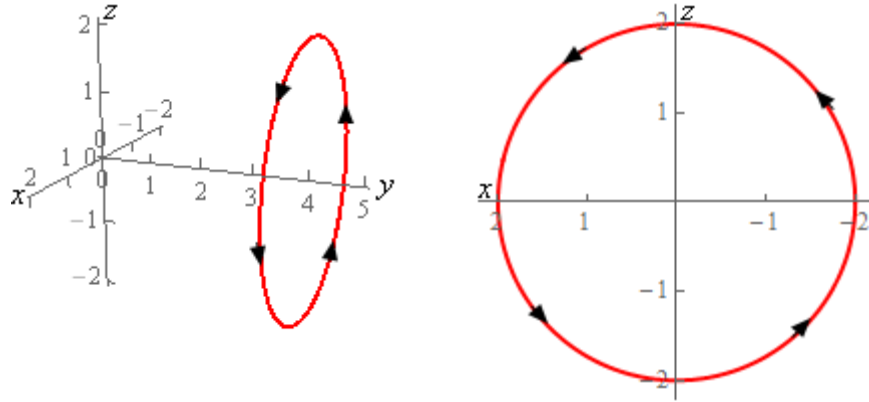
Vector Fields

1. Sketch the vector field for $\vec{F}(x, y) = 2x\vec{i} - 2\vec{j}$.
2. Sketch the vector field for $\vec{F}(x, y) = (y-1)\vec{i} + (x+y)\vec{j}$.
3. Compute the gradient vector field for $f(x, y) = y^2 \cos(2x - y)$.
4. Compute the gradient vector field for $f(x, y, z) = z^2 e^{x^2+4y} + \ln\left(\frac{xy}{z}\right)$.

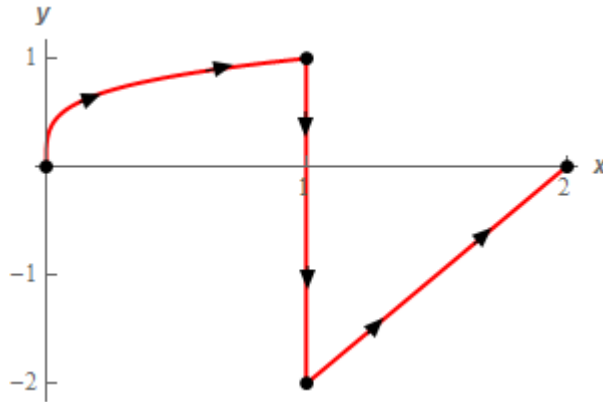
Line Integrals - Part I

For problems 1 – 7 evaluate the given line integral. Follow the direction of C as given in the problem statement.

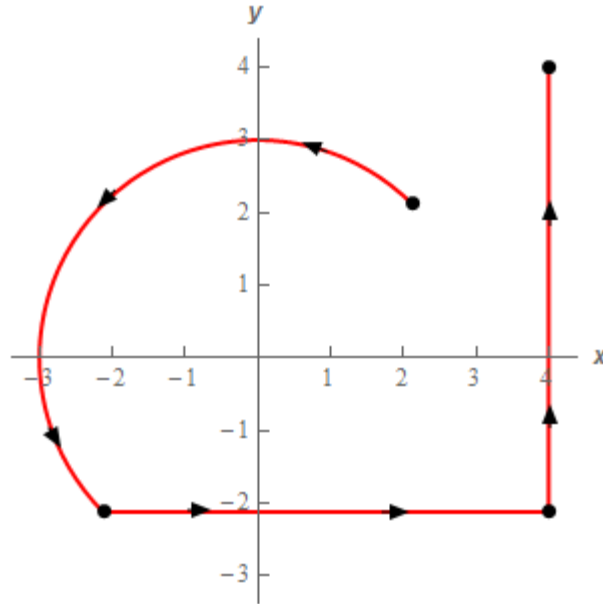
1. Evaluate $\int_C 3x^2 - 2y \, ds$ where C is the line segment from $(3, 6)$ to $(1, -1)$.
2. Evaluate $\int_C 2yx^2 - 4x \, ds$ where C is the lower half of the circle centered at the origin of radius 3 with clockwise rotation.
3. Evaluate $\int_C 6x \, ds$ where C is the portion of $y = x^2$ from $x = -1$ to $x = 2$. The direction of C is in the direction of increasing x .
4. Evaluate $\int_C xy - 4z \, ds$ where C is the line segment from $(1, 1, 0)$ to $(2, 3, -2)$.
5. Evaluate $\int_C x^2 y^2 \, ds$ where C is the circle centered at the origin of radius 2 centered on the y -axis at $y = 4$. See the sketches below for orientation. Note the “odd” axis orientation on the 2D circle is intentionally that way to match the 3D axis the direction.



6. Evaluate $\int_C 16y^5 ds$ where C is the portion of $x = y^4$ from $y = 0$ to $y = 1$ followed by the line segment from $(1,1)$ to $(1,-2)$ which in turn is followed by the line segment from $(1,-2)$ to $(2,0)$. See the sketch below for the direction.



7. Evaluate $\int_C 4y - x ds$ where C is the upper portion of the circle centered at the origin of radius 3 from $(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$ to $(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}})$ in the counter clockwise rotation followed by the line segment from $(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}})$ to $(4, -\frac{3}{\sqrt{2}})$ which in turn is followed by the line segment from $(4, -\frac{3}{\sqrt{2}})$ to $(4,4)$. See the sketch below for the direction.



8. Evaluate $\int_C y^3 - x^2 ds$ for each of the following curves.

- (a) C is the line segment from $(3, 6)$ to $(0, 0)$ followed by the line segment from $(0, 0)$ to $(3, -6)$.
- (b) C is the line segment from $(3, 6)$ to $(3, -6)$.

9. Evaluate $\int_C 4x^2 ds$ for each of the following curves.

- (a) C is the portion of the circle centered at the origin of radius 2 in the 1st quadrant rotating in the clockwise direction.
- (b) C is the line segment from $(0, 2)$ to $(2, 0)$.

10. Evaluate $\int_C 2x^3 ds$ for each of the following curves.

- (a) C is the portion $y = x^3$ from $x = -1$ to $x = 2$.
- (b) C is the portion $y = x^3$ from $x = 2$ to $x = -1$.

Line Integrals - Part II

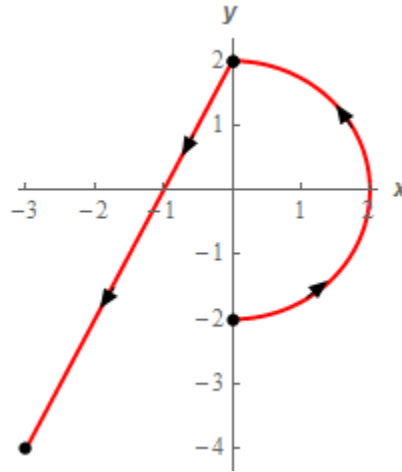
For problems 1 – 7 evaluate the given line integral. Follow the direction of C as given in the problem statement.

1. Evaluate $\int_C \sqrt{1+y} dy$ where C is the portion of $y = e^{2x}$ from $x = 0$ to $x = 2$.

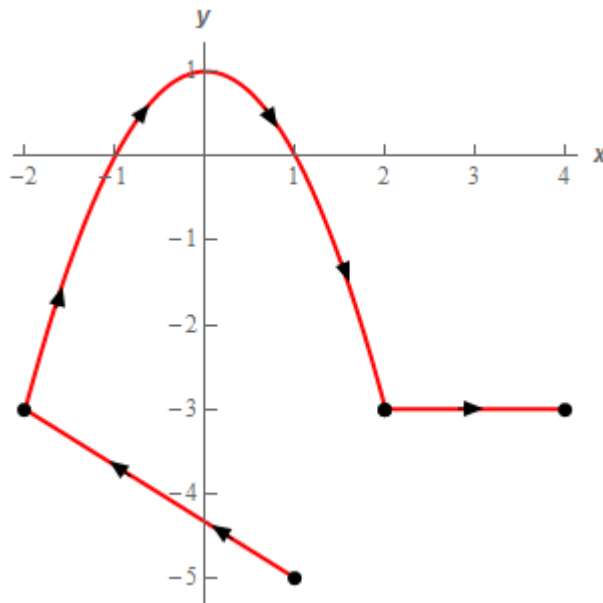
2. Evaluate $\int_C 2y dx + (1-x) dy$ where C is portion of $y = 1 - x^3$ from $x = -1$ to $x = 2$.

3. Evaluate $\int_C x^2 dy - yz dz$ where C is the line segment from $(4, -1, 2)$ to $(1, 7, -1)$.

4. Evaluate $\int_C 1 + x^3 dx$ where C is the right half of the circle of radius 2 with counter clockwise rotation followed by the line segment from $(0, 2)$ to $(-3, -4)$. See the sketch below for the direction.



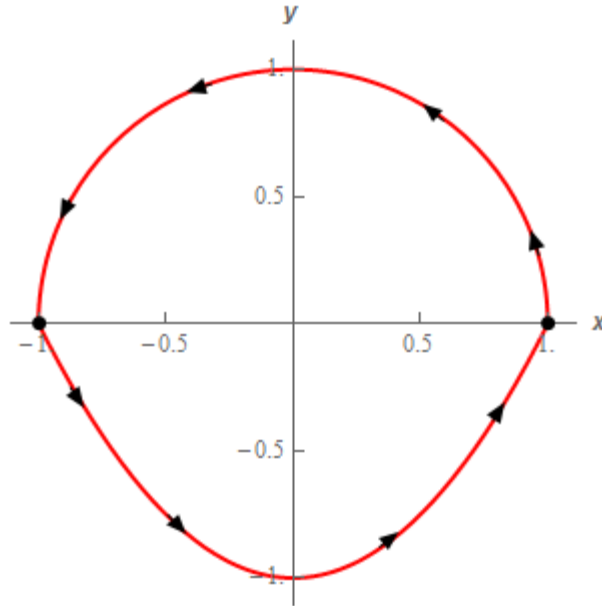
5. Evaluate $\int_C 2x^2 dy - xy dx$ where C is the line segment from $(1, -5)$ to $(-2, -3)$ followed by the portion of $y = 1 - x^2$ from $x = -2$ to $x = 2$ which in turn is followed by the line segment from $(2, -3)$ to $(4, -3)$. See the sketch below for the direction.



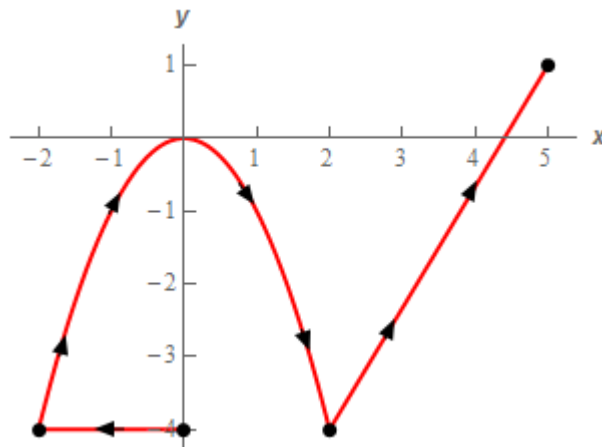
6. Evaluate $\int_C (x - y) dx - yx^2 dy$ for each of the following curves.
- (a) C is the portion of the circle of radius 6 in the 1st, 2nd and 3rd quadrant with clockwise rotation.
- (b) C is the line segment from $(0, -6)$ to $(6, 0)$.
7. Evaluate $\int_C x^3 dy - (y + 1) dx$ for each of the following curves.
- (a) C is the line segment from $(1, 7)$ to $(-2, 4)$.
- (b) C is the line segment from $(-2, 4)$ to $(1, 7)$.

Line Integrals of Vector Fields

1. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = y^2 \vec{i} + (3x - 6y) \vec{j}$ and C is the line segment from $(3, 7)$ to $(0, 12)$.
2. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = (x + y) \vec{i} + (1 - x) \vec{j}$ and C is the portion of $\frac{x^2}{4} + \frac{y^2}{9} = 1$ that is in the 4th quadrant with the counter clockwise rotation.
3. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = y^2 \vec{i} + (x^2 - 4) \vec{j}$ and C is the portion of $y = (x - 1)^2$ from $x = 0$ to $x = 3$.
4. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = e^{2x} \vec{i} + z(y + 1) \vec{j} + z^3 \vec{k}$ and C is given by $\vec{r}(t) = t^3 \vec{i} + (1 - 3t) \vec{j} + e^t \vec{k}$ for $0 \leq t \leq 2$.
5. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = 3y \vec{i} + (x^2 - y) \vec{j}$ and C is the upper half of the circle centered at the origin of radius 1 with counter clockwise rotation and the portion of $y = x^2 - 1$ from $x = -1$ to $x = 1$. See the sketch below.



6. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = xy\vec{i} + (1+3y)\vec{j}$ and C is the line segment from $(0, -4)$ to $(-2, -4)$ followed by portion of $y = -x^2$ from $x = -2$ to $x = 2$ which is in turn followed by the line segment from $(2, -4)$ to $(5, 1)$. See the sketch below.

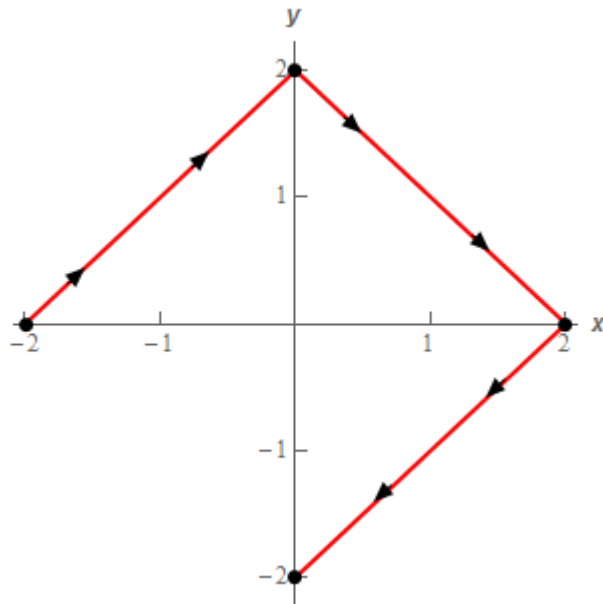


7. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = (6x - 2y)\vec{i} + x^2\vec{j}$ for each of the following curves.
- C is the line segment from $(6, -3)$ to $(0, 0)$ followed by the line segment from $(0, 0)$ to $(6, 3)$.
 - C is the line segment from $(6, -3)$ to $(6, 3)$.
8. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = 3\vec{i} + (xy - 2x)\vec{j}$ for each of the following curves.
- C is the upper half of the circle centered at the origin of radius 4 with counter

- clockwise rotation.
 (b) C is the upper half of the circle centered at the origin of radius 4 with clockwise rotation.

Fundamental Theorem for Line Integrals

1. Evaluate $\int_C \nabla f \cdot d\vec{r}$ where $f(x, y) = x^3(3 - y^2) + 4y$ and C is given by $\vec{r}(t) = \langle 3 - t^2, 5 - t \rangle$ with $-2 \leq t \leq 3$.
2. Evaluate $\int_C \nabla f \cdot d\vec{r}$ where $f(x, y) = ye^{x^2-1} + 4x\sqrt{y}$ and C is given by $\vec{r}(t) = \langle 1 - t, 2t^2 - 2t \rangle$ with $0 \leq t \leq 2$.
3. Given that $\int_C \vec{F} \cdot d\vec{r}$ is independent of path compute $\int_C \vec{F} \cdot d\vec{r}$ where C is the ellipse given by $\frac{(x-5)^2}{4} + \frac{y^2}{9} = 1$ with the counter clockwise rotation.
4. Evaluate $\int_C \nabla f \cdot d\vec{r}$ where $f(x, y) = e^{xy} - x^2 + y^3$ and C is the curve shown below.



Conservative Vector Fields

For problems 1 – 3 determine if the vector field is conservative.

$$1. \vec{F} = (x^3 - 4xy^2 + 2)\vec{i} + (6x - 7y + x^3y^3)\vec{j}$$

$$2. \vec{F} = (2x \sin(2y) - 3y^2)\vec{i} + (2 - 6xy + 2x^2 \cos(2y))\vec{j}$$

$$3. \vec{F} = (6 - 2xy + y^3)\vec{i} + (x^2 - 8y + 3xy^2)\vec{j}$$

For problems 4 – 8 find the potential function for the vector field.

$$4. \vec{F} = \left(6x^2 - 2xy^2 + \frac{y}{2\sqrt{x}}\right)\vec{i} - (2x^2y - 4 - \sqrt{x})\vec{j}$$

$$5. \vec{F} = y^2(1 + \cos(x + y))\vec{i} + (2xy - 2y + y^2 \cos(x + y) + 2y \sin(x + y))\vec{j}$$

$$6. \vec{F} = (2z^4 - 2y - y^3)\vec{i} + (z - 2x - 3xy^2)\vec{j} + (6 + y + 8xz^3)\vec{k}$$

$$7. \vec{F} = \frac{2xy}{z^3}\vec{i} + \left(2y - z^2 + \frac{x^2}{z^3}\right)\vec{j} - \left(4z^3 + 2yz + \frac{3x^2y}{z^4}\right)\vec{k}$$

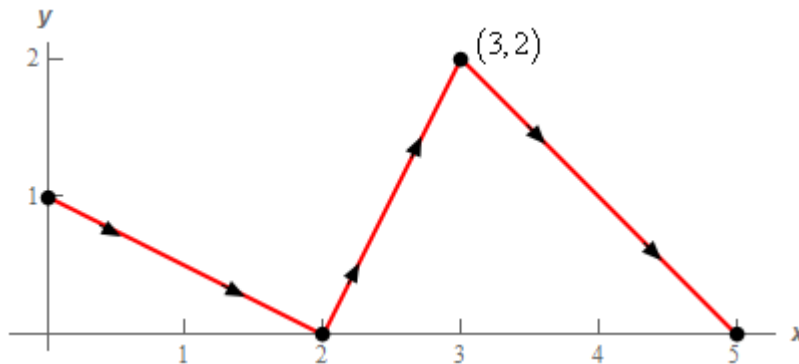
8. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the portion of the circle centered at the origin with radius 2 in the

1st quadrant with counter clockwise rotation and

$$\vec{F}(x, y) = (2xy - 4 - \frac{1}{2} \sin(\frac{1}{2}x) \sin(\frac{1}{2}y))\vec{i} + (x^2 + \frac{1}{2} \cos(\frac{1}{2}x) \cos(\frac{1}{2}y))\vec{j}.$$

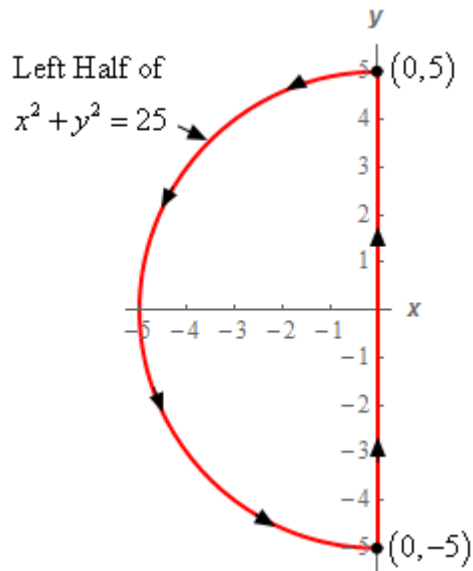
9. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = (2ye^{xy} + 2xe^{x^2-y^2})\vec{i} + (2xe^{xy} - 2ye^{x^2-y^2})\vec{j}$ and C is

the curve shown below.

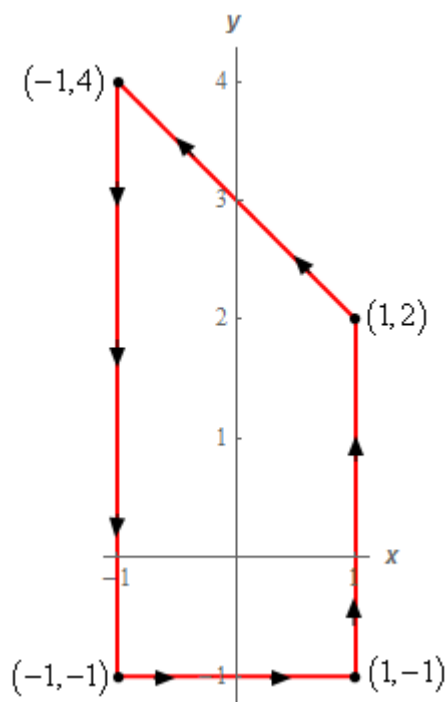


Green's Theorem

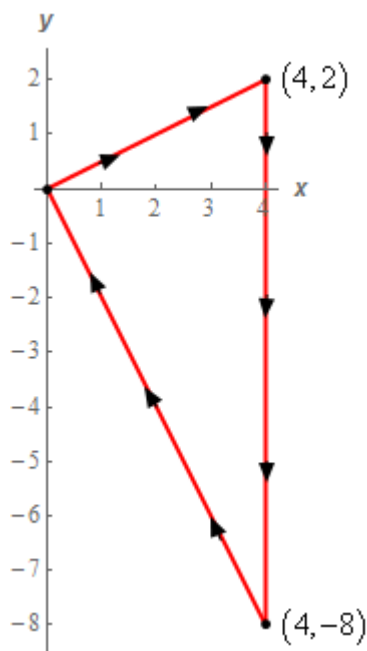
1. Use Green's Theorem to evaluate $\int_C yx^2 dx - x^2 dy$ where C is shown below.



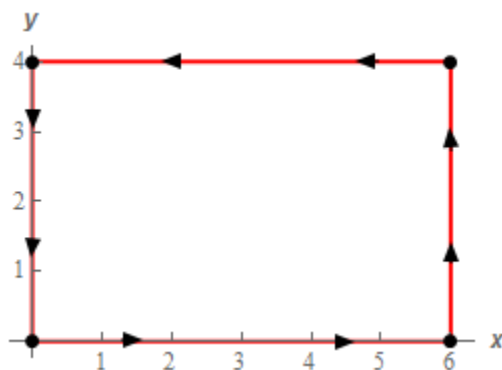
2. Use Green's Theorem to evaluate $\int_C (6y - 9x) dy - (yx - x^3) dx$ where C is shown below.



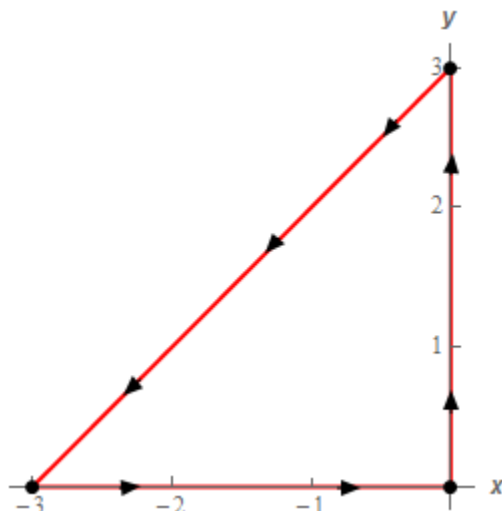
3. Use Green's Theorem to evaluate $\int_C x^2 y^2 dx + (yx^3 + y^2) dy$ where C is shown below.



4. Use Green's Theorem to evaluate $\int_C (y^4 - 2y) dx - (6x - 4xy^3) dy$ where C is shown below.



5. Verify Green's Theorem for $\oint_C (xy^2 + x^2) dx + (4x - 1) dy$ where C is shown below by **(a)** computing the line integral directly and **(b)** using Green's Theorem to compute the line integral.



Curl and Divergence

For problems 1 & 2 compute $\text{div } \vec{F}$ and $\text{curl } \vec{F}$.

1. $\vec{F} = x^2 y \vec{i} - (z^3 - 3x) \vec{j} + 4y^2 \vec{k}$
2. $\vec{F} = (3x + 2z^2) \vec{i} + \frac{x^3 y^2}{z} \vec{j} - (z - 7x) \vec{k}$

For problems 3 & 4 determine if the vector field is conservative.

3. $\vec{F} = \left(4y^2 + \frac{3x^2 y}{z^2} \right) \vec{i} + \left(8xy + \frac{x^3}{z^2} \right) \vec{j} + \left(11 - \frac{2x^3 y}{z^3} \right) \vec{k}$
4. $\vec{F} = 6x \vec{i} + (2y - y^2) \vec{j} + (6z - x^3) \vec{k}$

Surface Integrals

Introduction

Here are a set of practice problems for the Surface Integrals chapter of my Calculus III notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

16. If you'd like a pdf document containing the solutions go to the note page for the section you'd like solutions for and select the download solutions link from there. Or,
17. Go to the download page for the site <http://tutorial.math.lamar.edu/download.aspx> and select the section you'd like solutions for and a link will be provided there.
18. If you'd like to view the solutions on the web or solutions to an individual problem you can go to the problem set web page, select the problem you want the solution for. At this point I do not provide pdf versions of individual solutions, but for a particular problem you can select "Printable View" from the "Solution Pane Options" to get a printable version.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of topics in this chapter that have practice problems written for them.

[Parametric Surfaces](#)
[Surface Integrals](#)
[Surface Integrals of Vector Fields](#)
[Stokes' Theorem](#)
[Divergence Theorem](#)

Parametric Surfaces

For problems 1 – 6 write down a set of parametric equations for the given surface.

1. The plane $7x + 3y + 4z = 15$.
2. The portion of the plane $7x + 3y + 4z = 15$ that lies in the 1st octant.
3. The cylinder $x^2 + y^2 = 5$ for $-1 \leq z \leq 6$.
4. The portion of $y = 4 - x^2 - z^2$ that is in front of $y = -6$.
5. The portion of the sphere of radius 6 with $x \geq 0$.
6. The tangent plane to the surface given by the following parametric equation at the point $(8, 14, 2)$.

$$\vec{r}(u, v) = (u^2 + 2u)\vec{i} + (3v - 2u)\vec{j} + (6v - 10)\vec{k}$$
7. Determine the surface area of the portion of $2x + 3y + 6z = 9$ that is inside the cylinder $x^2 + y^2 = 7$.
8. Determine the surface area of the portion of $x^2 + y^2 + z^2 = 25$ with $z \leq 0$.

9. Determine the surface area of the portion of $z = 3 + 2y + \frac{1}{4}x^4$ that is above the region in the xy -plane bounded by $y = x^5$, $x = 1$ and the y -axis.

10. Determine the surface area of the portion of the surface given by the following parametric equation that lies inside the cylinder $u^2 + v^2 = 4$.

$$\vec{r}(u, v) = \langle 2u, vu, 1 - 2v \rangle$$

Surface Integrals

1. Evaluate $\iint_S z + 3y - x^2 \, dS$ where S is the portion of $z = 2 - 3y + x^2$ that lies over the triangle in the xy -plane with vertices $(0, 0)$, $(2, 0)$ and $(2, -4)$.

2. Evaluate $\iint_S 40y \, dS$ where S is the portion of $y = 3x^2 + 3z^2$ that lies behind $y = 6$.

3. Evaluate $\iint_S 2y \, dS$ where S is the portion of $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$.

4. Evaluate $\iint_S xz \, dS$ where S is the portion of the sphere of radius 3 with $x \leq 0$, $y \geq 0$ and $z \geq 0$.

5. Evaluate $\iint_S yz + 4xy \, dS$ where S is the surface of the solid bounded by $4x + 2y + z = 8$, $z = 0$, $y = 0$ and $x = 0$. Note that all four surfaces of this solid are included in S .

6. Evaluate $\iint_S x - z \, dS$ where S is the surface of the solid bounded by $x^2 + y^2 = 4$, $z = x - 3$, and $z = x + 2$. Note that all three surfaces of this solid are included in S .

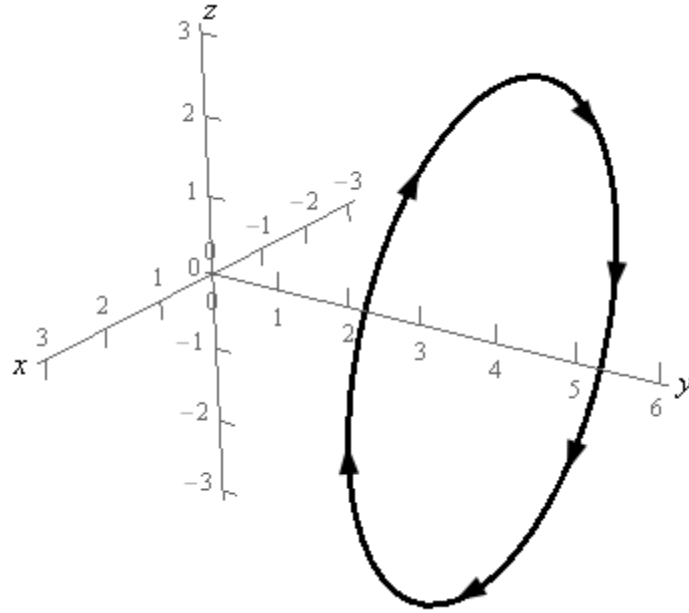
Surface Integrals of Vector Fields

1. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = 3x\vec{i} + 2z\vec{j} + (1 - y^2)\vec{k}$ and S is the portion of $z = 2 - 3y + x^2$ that lies over the triangle in the xy -plane with vertices $(0, 0)$, $(2, 0)$ and $(2, -4)$ oriented in the negative z -axis direction.

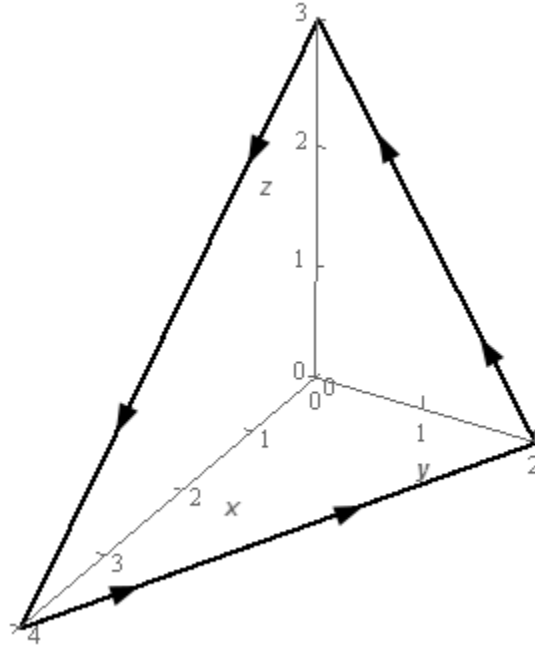
2. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = -x\vec{i} + 2y\vec{j} - z\vec{k}$ and S is the portion of $y = 3x^2 + 3z^2$ that lies behind $y = 6$ oriented in the positive y -axis direction.
3. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = x^2\vec{i} + 2z\vec{j} - 3y\vec{k}$ and S is the portion of $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$ oriented outwards (*i.e.* away from the x -axis).
4. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \vec{i} + z\vec{j} + 6x\vec{k}$ and S is the portion of the sphere of radius 3 with $x \leq 0$, $y \geq 0$ and $z \geq 0$ oriented inward (*i.e.* towards the origin).
5. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = y\vec{i} + 2x\vec{j} + (z - 8)\vec{k}$ and S is the surface of the solid bounded by $4x + 2y + z = 8$, $z = 0$, $y = 0$ and $x = 0$ with the positive orientation. Note that all four surfaces of this solid are included in S .
6. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = yz\vec{i} + x\vec{j} + 3y^2\vec{k}$ and S is the surface of the solid bounded by $x^2 + y^2 = 4$, $z = x - 3$, and $z = x + 2$ with the negative orientation. Note that all three surfaces of this solid are included in S .

Stokes' Theorem

1. Use Stokes' Theorem to evaluate $\iint_S \text{curl } \vec{F} \cdot d\vec{S}$ where $\vec{F} = y\vec{i} - x\vec{j} + yx^3\vec{k}$ and S is the portion of the sphere of radius 4 with $z \geq 0$ and the upwards orientation.
2. Use Stokes' Theorem to evaluate $\iint_S \text{curl } \vec{F} \cdot d\vec{S}$ where $\vec{F} = (z^2 - 1)\vec{i} + (z + xy^3)\vec{j} + 6\vec{k}$ and S is the portion of $x = 6 - 4y^2 - 4z^2$ in front of $x = -2$ with orientation in the negative x -axis direction.
3. Use Stokes' Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = -yz\vec{i} + (4y + 1)\vec{j} + xy\vec{k}$ and C is the circle of radius 3 at $y = 4$ and perpendicular to the y -axis. C has a clockwise rotation if you are looking down the y -axis from the positive y -axis to the negative y -axis. See the figure below for a sketch of the curve.



4. Use Stokes' Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (3yx^2 + z^3)\vec{i} + y^2\vec{j} + 4yx^2\vec{k}$ and C is is triangle with vertices $(0, 0, 3)$, $(0, 2, 0)$ and $(4, 0, 0)$. C has a counter clockwise rotation if you are above the triangle and looking down towards the xy -plane. See the figure below for a sketch of the curve.



Divergence Theorem

1. Use the Divergence Theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = yx^2 \vec{i} + (xy^2 - 3z^4) \vec{j} + (x^3 + y^2) \vec{k}$

and S is the surface of the sphere of radius 4 with $z \leq 0$ and $y \leq 0$. Note that all three surfaces of this solid are included in S .

2. Use the Divergence Theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \sin(\pi x) \vec{i} + zy^3 \vec{j} + (z^2 + 4x) \vec{k}$ and S

is the surface of the box with $-1 \leq x \leq 2$, $0 \leq y \leq 1$ and $1 \leq z \leq 4$. Note that all six sides of the box are included in S .

3. Use the Divergence Theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = 2xz \vec{i} + (1 - 4xy^2) \vec{j} + (2z - z^2) \vec{k}$ and

S is the surface of the solid bounded by $z = 6 - 2x^2 - 2y^2$ and the plane $z = 0$. Note that both of the surfaces of this solid included in S .