Vector Calculus 20E, Fall 2019, Second Midterm

Fifty minutes, three problems, no calculators. Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Let γ be the quarter-circle inside $\mathbb{R}^2$ given by $x^2+y^2=4, x \geq 0, y \geq 0$ and oriented anticlockwise. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x,y) = (xy, x^2y^2)$. Compute

$$\int_\gamma \mathbf{F} \cdot \mathrm{d}\mathbf{s}.$$

2. Let Σ be the surface in $\mathbb{R}^3$ given by $x+y+z=1, x \geq 0, y \geq 0, z \geq 0$ and let f be the function $f(x,y,z) = x^2$. Compute the average value of f over Σ.

3. Let Σ be the piece of the paraboloid $z = x^2+y^2$ given by $x \geq 0, y \geq 0, z \leq 1$, oriented by saying that the positive z-axis is an example of a normal vector pointing in the positive direction. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x,y,z) = (x,y,z)$. Compute

$$\int_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$

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4. Let γ be the quarter-circle inside $\mathbb{R}^2$ given by $x^2+y^2=4, x \geq 0, y \geq 0$ and oriented anticlockwise. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x,y) = (xy, x^2y^2)$. Compute

$$\int_\gamma \mathbf{F} \cdot \mathrm{d}\mathbf{s}.$$

5. Let Σ be the surface in $\mathbb{R}^3$ given by $x+y+z=1, x \geq 0, y \geq 0, z \geq 0$ and let f be the function $f(x,y,z) = x^2$. Compute the average value of f over Σ.

6. Let Σ be the piece of the paraboloid $z = x^2+y^2$ given by $x \geq 0, y \geq 0, z \leq 1$, oriented by saying that the positive z-axis is an example of a normal vector pointing in the positive direction. Let $\mathbf{F}$ be the vector field given by $\mathbf{F}(x,y,z) = (x,y,z)$. Compute

$$\int_\Sigma \mathbf{F} \cdot \mathrm{d}\mathbf{A}.$$