

HOMEWORK 4, DUE WEDNESDAY FEBRUARY 5TH

1. Find all topologies on the set

$$X = \{a, b, c, d\}$$

up to homeomorphism. (This means, give a list of topologies on X , such that every other topology on X is homeomorphic to exactly one topology in your list).

2. Let X and Y be two sets. Give both sets the topology where the closed sets are the finite sets, plus the whole set. Under what conditions are X and Y homeomorphic?

3. Show that any two closed and bounded intervals in $\mathbb{R}$ are homeomorphic.

4. Complete the proof of Theorem 7.2.

5. True or false? If true then give a proof and if false then give a counterexample.

(i) If $(X, \mathcal{T})$ and $(Y, \mathcal{S})$ are topological spaces then $X \times Y$ and $Y \times X$, both with the product topology, are homeomorphic.

(ii) Let $\mathcal{T}$ be the Euclidean topology on $\mathbb{R}$ and let $\mathcal{S}$ be the topology where the open sets are half open intervals of the form (a, ∞) . Then $(\mathbb{R}, \mathcal{T})$ and $(\mathbb{R}, \mathcal{S})$ are homeomorphic.

(iii) Let $\mathcal{T}$ be the topology where the open sets are half open intervals of the form (a, ∞) and let $\mathcal{S}$ be the topology where the open sets are half open intervals of the form $(-\infty, a)$. Then $(\mathbb{R}, \mathcal{T})$ and $(\mathbb{R}, \mathcal{S})$ are homeomorphic.

(iv) If $(X, \mathcal{T})$ and $(Y, \mathcal{S})$ are homeomorphic topological spaces then $(X, \mathcal{T})$ is Hausdorff if and only if $(Y, \mathcal{S})$ is Hausdorff.

(v) If we are given four topological spaces, X , Y , Z and W and $X \times Y$ is homeomorphic to $Z \times W$ then X is homeomorphic to Z or to W .

Challenge Problems: (Just for fun)

6. (i) Show that one can find uncountably many pairwise disjoint circles in $\mathbb{R}^2$.

(ii) Show that one cannot find uncountably many pairwise disjoint discs in $\mathbb{R}^2$.