HOMEWORK 8, DUE THURSDAY MARCH 7TH

1. Let M, N and P be R-modules and let F be a free R-module of rank n. Show that there are isomorphisms, which are all natural (except for the last):
(a)
$$M \underset{R}{\otimes} N \simeq N \underset{R}{\otimes} M.$$
(b)
$$M \underset{R}{\otimes} (N \underset{R}{\otimes} P) \simeq (M \underset{R}{\otimes} N) \underset{R}{\otimes} P.$$
(c)
$$R \underset{R}{\otimes} M \simeq M.$$
(d)
$$M \underset{R}{\otimes} (N \oplus P) \simeq (M \underset{R}{\otimes} N) \oplus (M \underset{R}{\otimes} P).$$
(e)
$$F \underset{R}{\otimes} M \simeq M^n,$$
the direct sum of copies of M with itself n times.
2. Let m and n be integers. Identify $\mathbb{Z}_m \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_n$.
3. Show that if M and N are two finitely generated (respectively Noetherian) R-modules (respectively and R is Noetherian) then so is $M \underset{R}{\otimes} N$.

Challenge Problems:

4. If G and H are two finitely generated abelian groups, show how to determine the tensor product
$$G \underset{\mathbb{Z}}{\otimes} H.$$
5. Show that
$$\mathbb{Q}/\mathbb{Z} \underset{\mathbb{Z}}{\otimes} \mathbb{Q}/\mathbb{Z} \simeq 0.$$
6. Show that if M and N are two Noetherian R-modules then so is $M \underset{R}{\otimes} N$.
7. Let M and N be two R-modules. Show that
$$\bigwedge^n (M \oplus N) \qquad \text{and} \qquad \bigoplus_{p+q=n} \wedge^p M \oplus \wedge^q N$$
are isomorphic.