HOMEWORK 2, DUE THURSDAY JANUARY 25TH

1. Chapter 4, Section 3: 3, 4, 5, 6, 9, 12, 14, 15, 18, 19, 20, 22, 23, 24.

Challenge Problems: (Just for fun)

2. Chapter 4, Section 3: 26, 27.
3. Let $\mathbb{F}_2$ be the field with two elements and let

$$G = \mathrm{GL}_3(\mathbb{F}_2)$$

be the group of all 3×3 invertible matrices with entries in $\mathbb{F}_2$.
(i) Show that G acts on the three dimensional vector space

$$\mathbb{F}_2^3 = \mathbb{F}_2 \oplus \mathbb{F}_2 \oplus \mathbb{F}_2,$$

the set of vectors (a, b, c) with entries in a, b, $c\mathbb{F}_2$.
(ii) Show that the vector space minus the origin

$$\mathbb{F}_2^3 \setminus \{(0, 0, 0)\}.$$

coincides

$$\mathbb{P}^2_{\mathbb{F}_2}$$

which is the set of lines (through the origin) in

$$\mathbb{F}_2^3.$$

(iii) Show that

$$\mathbb{P}^2_{\mathbb{F}_2}$$

has seven elements.
(iv) Show that G has 168 elements.
(v) Show that G is a simple group.
(vi) We say that l is a projective line in

$$\mathbb{P}^2_{\mathbb{F}_2}$$

if it is the set of lines in $\mathbb{F}_2^3$ contained in a plane (a two dimensional linear subspace of $\mathbb{F}_2^3$).
Show that there are seven projective lines.
(vii) Show that

$$\mathbb{P}^2_{\mathbb{F}_2}$$

is a projective plane, in the sense that two distinct points determine a unique line and two distinct lines determine a unique point.