HOMEWORK 9, DUE THURSDAY DECEMBER 7TH

1. For Chapter 2, Section 11: 1, 2, 3, 4, 5, 10, 16, 17.
2. Let $K \subset H \subset G$ be three groups.
True or False? If true then give a proof and if false then give a counterexample.
(i) If K is characteristically normal in H and H is characteristically normal in G then K is characteristically normal in G.
(ii) If K is normal in H and H is characteristically normal in G then K is normal in G.
(iii) If K is characteristically normal in H and H is normal in G then K is normal in G.
(iv) If K is normal in H and H is normal in G then K is normal in G.
3. Show that the automorphism group of a cyclic group of order n is isomorphic to

$$\begin{cases} U_n & \text{if } n \text{ is finite} \\ \mathbb{Z}_2 & \text{otherwise.} \end{cases}$$

4. Classify all groups of order n, where $n \in \{11, 13, 14\}$.
Challenge Problems (Just for fun)
5. Complete the classification of all groups of order n, where $10 < n < 15$, that is, classify all groups of order $n = 12$.
6. Let G be a simple group of order n. If $60 < n < 168$ then show that n is prime.
7. Classify all finite subgroups H of $\mathrm{GL}(3, \mathbb{R})$. (*Hint: pick a non-zero vector V and consider its orbit under G).*
8. Let G be a group. We say that G is **Jordan** if there is a natural number k such that if H is any finite subgroup of G then there is an abelian subgroup $A \subset H$ whose index is at most k.
(i) Show that every finite group is Jordan.
(ii) Show that every subgroup of a Jordan group is Jordan.
(iii) Show that the product of two Jordan groups is Jordan.
(iv) Show that G is Jordan if and only if there is a natural number l such that if H is any finite subgroup of G then there is an abelian *normal* subgroup $A \triangleleft H$ whose index is at most l.
(v) Show that $\mathrm{GL}(3, \mathbb{R})$ is Jordan.
(vi) Find the optimal value of k from (v).