HOMEWORK 7, DUE WEDNESDAY NOVEMBER 22ND

1. For Chapter 2, Section 9: 1, 2, 3.
2. Let H and K be two normal subgroups of a group G, whose intersection is the trivial subgroup. Prove that every element of H commutes with every element of K. (*Hint. Consider the commutator of an element of* H *and an element of* K).
3. Prove that a group G is isomorphic to the product of two groups H' and K' if and only if G contains two normal subgroups H and K, such that

   (1) H is isomorphic to H' and K is isomorphic to K'.
   (2) $H \cap K = \{e\}$.
   (3) $G = H \vee K$.

Challenge Problems (Just for fun)
4. Let $\mathcal{C}$ be a category.
The **direct sum** of two objects X and Y is an object Z together with two morphisms $i\colon X \longrightarrow Z$ and $j\colon Y \longrightarrow Z$ which are universal amongst all such morphisms.
(i) Write out what it means for i and j to be universal, by drawing the appropriate commutative diagram with the induced morphism.
(ii) Identify the direct sum in the category of sets.
(iii) Identify the direct sum in the category of abelian groups.
In particular note that the direct sum depends heavily on the underlying category.