

FIFTH HOMEWORK

1. Let S be a smooth surface and let ν be any valuation. Show that the log discrepancy of ν with respect to K_S is at least two, with equality iff ν is the blow up of a point.
2. Let (X, Δ) be a log pair of log discrepancy at most zero. Define the anti-log discrepancy b to be the supremum of the log discrepancies of valuations of log discrepancy at most zero,

$$b = \sup_{a(\nu, X, \Delta) \leq 0} a(\nu, X, \Delta).$$

Show that the anti-log discrepancy is in fact a maximum, that is, show that there is a valuation ν such that $a(\nu, X, \Delta) = b$.

3. Let (X, Δ) be a log pair and let $D \geq 0$ be any divisor. The log canonical threshold of D with respect to (X, Δ) is the largest real number such that $(X, \Delta + D)$ has log discrepancy at least zero.

Let $C = C_n$ be the plane curve $(y^2 + x^n = 0) \subset \mathbb{C}^2$ and let $\lambda = \lambda_n$ be the log canonical threshold of C with respect to $(\mathbb{A}^2, 0)$, so that $(\mathbb{A}^2, \lambda C)$ has log discrepancy zero.

- (i) What is λ_2 ?
- (ii) What is λ_3 ?
- (iii) What is λ_4 ?
- (iv) Can you guess the log canonical threshold when $C = C_{n,m}$ is the plane curve given by $y^m + x^n$?
4. Let S be a surface over $\mathbb{C}$ which is invariant under complex conjugation (in other words, suppose that S is a surface over $\mathbb{R}$). Classify the K_S -extremal rays which are invariant under complex conjugation (in other words, classify the K_S -extremal rays of the real structure).