

SECOND HOMEWORK

1. Let $H \subset \mathbb{P}^n$ be a hypersurface of degree d . Identify the linear system $|H|$. Is $|H|$ base point free? What is its dimension?
2. Let C be a smooth curve and let $p \in C$ be a point of C . Then $h^0(C, \mathcal{O}_C(p)) > 1$ if and only if $C \simeq \mathbb{P}^1$.
3. Let C be a smooth curve of genus g and let D be a divisor of degree d . Show that if $d > 2g$ then D is very ample. Give examples to show that this bound is sharp. Show that D is ample if and only if $d > 0$.
4. Give an example of a morphism $f: X \rightarrow Y$ and a $\mathbb{Q}$ -Cartier divisor D on Y such that f^*D is semiample but D is not semiample.
5. Let X be a normal projective variety and let H be an ample $\mathbb{Q}$ -divisor. Show that a $\mathbb{Q}$ -Cartier divisor D is nef (that is $D \cdot C \geq 0$ for all curves C in X) if and only if $D + \epsilon H$ is ample for all $\epsilon > 0$. Conclude that D is nef if and only if $D^k \cdot V \geq 0$ for all subvarieties $V \subset X$ of dimension k .