PRACTICE PROBLEMS FOR THE FIRST MIDTERM

1. (a) Give the definition of:

(i) a complex number
(ii) the real part
(iii) the imaginary part
(iv) the complex conjugate
(v) the modulus
(vi) the argument
(vii) the principal value of the argument
(viii) the exponential function
(ix) the sine function
(x) the cosine function
(xi) a path
(xii) a closed path
(xiii) the unit circle
(xiv) an open disk
(xv) a closed disk
(xvi) the unit disk
(xvii) a region (no need to define open or connected)
(xviii) the extended complex plane
(xix) a Möbius transformation
(xx) the upper half plane
(xxi) a period
(xxii) the principal value of the logarithm
(xxiii) the harmonic series
(xxiv) the alternating harmonic series
(xxv) absolute convergence
(xxvi) conditional convergence.

(b) State

(i) the triangle inequality
(ii) the fundamental theorem of algebra
(iii) Euler's formula
(iv) DeMoivre's theorem.

2. Find formulas for

$$\cos 5\theta \qquad \text{and} \qquad \sin 5\theta,$$

involving only $\cos\theta$ and $\sin\theta$.

3. Let z and w be complex numbers.
(a) Show that

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \qquad \text{and} \qquad \sin z = \frac{e^{iz} - e^{-iz}}{2i}.$$

(b) Show that cos and sin are periodic functions with period 2π.
(c) Prove the addition formulas:

$$\begin{aligned} \cos(z+w) &= \cos z \cos w - \sin z \sin w \\ \sin(z+w) &= \cos z \sin w + \sin z \cos w. \end{aligned}$$

4. Write down a Möbius transformation that takes -2 to $1-2i$, i to 0 and 2 to $1+2i$.
5. Find a power series expansion of

$$\frac{1}{2-3z}$$

centred at $-i$. What is the radius of convergence?