HOMEWORK 9, DUE FRIDAY MARCH 13TH, 12PM

1. Find the Laurent expansion centred at $z = -1$ which converges at $z = 1/2$ for the following two functions

(a)
$$\frac{1}{z^2 - z}.$$

(b)
$$\frac{z - 1}{z + 1}.$$

2. Find the residue at $z = 0$ of

(a)
$$\frac{1}{z + z^2}.$$

(b)
$$z \cos\left(\frac{1}{z}\right).$$

(c)
$$\frac{\sinh z}{z^4(1 - z^2)}.$$

3. Suppose that a is an isolated singularity of $f(z)$ and that $(z-a)^n f(z)$ is bounded near a. Show that either a is removable or that $f(z)$ has a pole of order at most n.

4. Evaluate

(a)
$$\oint_{|z|=2} \frac{z}{\cos z}\, dz.$$

(b)
$$\oint_{|z|=3} \frac{e^{-z}}{z^2}\, dz.$$

(c)
$$\oint_{|z|=1} z^2 e^{1/z}\, dz.$$

Challenge Problems: (Just for fun)