

ROUGH OUTLINE OF LECTURES 7 AND 8

We will go through the calculation of the Dirichlet integral

$$\int_0^{\infty} \frac{\sin x}{x} dx.$$

This will take up Lecture 7 and probably spill into Lecture 8. This material is covered in §89 of the book but we take a slightly different approach.

The Dirichlet integral is not absolutely convergent for two reasons. The integral of $1/x$ diverges at both ∞ and at 0 .

So first of all we need a refinement of the Cauchy principal value which takes account of finite singularities. Again the model is what happens when we compute the contour integral.

Secondly we need to use indented paths. We integrate

$$f(z) = \frac{e^{iz}}{z}$$

around a small perturbation of the standard contour which takes a small detour around 0 , a small semicircle.

After that we will use indented paths to calculate integrals which involve branch points, see §90. We will then move onto integrals along a branch cut, §91. Recall that to define certain holomorphic functions, such as \sqrt{z} and $\text{Log } z$, we need to cut the plane along curves, we need to make a branch cut.