

**TAKE HOME MIDTERM EXAM
MATH 120B, UCSD, SPRING 20**

You have 24 hours.

There are 6 problems, and the total number of points is 100.

Please make your work as clear and easy to follow as possible. There is no need to be verbose but explain all of the steps, using your own words. You may consult the lecture notes and model answers but you may not use any other reference nor may you confer with anyone. You may use any of the standard results in the lecture notes as long as you clearly state what you are using. If you don't know how to solve the whole problem answer the portion you can solve.

Please submit your answers on Gradescope by 5pm on Thursday April 30th.

1. (3pts) Give three reasons why one should not cheat on this exam.
2. (17pts) Calculate

$$\int_{-\infty}^{\infty} \frac{1+x}{1+x^3} dx.$$

3. (20pts) Calculate

$$\int_0^{2\pi} \frac{d\theta}{(\cos \theta)^4 + (\sin \theta)^4}.$$

4. (20pts) Calculate

$$\int_{-\infty}^{\infty} \frac{x^3 \sin x}{(1+x^2)^2} dx.$$

5. (20pts) Calculate

$$\int_0^{\infty} \frac{(\ln x)^2}{(x^2+1)^2} dx.$$

6. (20pts) Calculate the Cauchy principal value of

$$\int_0^{\infty} \frac{x^{a-1}}{(x+b)(x+c)} dx \quad \text{where} \quad a \in (0,1), b < 0, c < 0.$$

7. (Extra credit: 10pts) If $p(z)$ is a polynomial of degree at least two then show that the sum of the residues of the rational function

$$\frac{1}{p(z)}$$

is zero.

8. (Extra credit: 20pts) Calculate

$$\int_{-\infty}^{\infty} \frac{1}{1+x+x^2+\dots+x^{2m}} dx.$$

for every natural number m . What is the limit as m goes to infinity?