HOMEWORK 4, DUE WEDNESDAY APRIL 29TH, 12PM

1. Show that $z^4 + 2z^2 - z + 1$ has exactly one root in each quadrant.
2. How many roots does $z^4 + z^3 + 4z^2 + 3z + 2$ have in each quadrant?
3. Find the number of roots of $z^6 + 4z^4 + z^3 + 2z^2 + z + 5$ in the first quadrant.
4. Let α be a real number. Find the number of roots of $z^4 + z^3 + 4z^2 + \alpha z + 3$ such that $\mathrm{Re}(z) < 0$.
5. Show that $2z^5 + 6z - 1$ has one root in the interval $(0, 1)$ and four roots in the annulus

$$\{ z \in \mathbb{C} \mid 1 < |z| < 2 \}.$$

6. Show that if m and n are positive integers then the polynomial

$$p(z) = 1 + z + \frac{z^2}{2!} + \cdots + \frac{z^m}{m!} + 3z^n$$

has exactly n roots in the unit disk Δ.
7. If λ is a complex number such that $|\lambda| < 1$, then

$$(z - 1)^n e^z + \lambda(z + 1)^n$$

has n zeroes in the right hand plane, $\mathrm{Re}(z) > 0$. Show these zeroes are simple if $\lambda \neq 0$.

Challenge Problems: (Just for fun)

8. Let U be a bounded domain and let $f(z)$ and $h(z)$ be two meromorphic functions on U that are holomorphic on ∂U. Suppose that

$$|h(z)| < |f(z)|$$

on ∂U.
(i) Give an example where $f(z)$ and $f(z)+h(z)$ have a different number of zeroes on U.
(ii) What comparison can we make between $f(z)$ and $f(z) + h(z)$? Prove your assertion.