HOMEWORK 2, DUE WEDNESDAY APRIL 15TH, 12PM

-1. Let $a \in \mathbb{C}$. Suppose that

$$f(z) = \frac{p(z)}{q(z)}$$

where $p(z)$ and $q(z)$ are holomorphic at a, a is not a zero of $p(z)$ and a is a simple zero of $q(z)$.
Show that $f(z)$ has an isolated singularity at a, a simple pole and that

$$\operatorname{Res}_a f(z) = \frac{p(a)}{q'(a)}.$$

0. Show that

$$\int_{\gamma_2} |e^{aiz}||\mathrm{d}z| < \frac{\pi}{a},$$

where γ_2 is any semicircle in the upper half plane, centred at the origin and $a > 0$.
1. Show that

$$\int_{-\infty}^{\infty} \frac{x^3 \sin ax}{x^4 + 4}\,\mathrm{d}x = \pi e^{-a} \cos a \qquad \text{where} \qquad a > 0.$$

2. Calculate the Cauchy principal value of

$$\int_{-\infty}^{\infty} \frac{x \sin x \mathrm{d}x}{x^2 + 2x + 2}.$$

3. Question 12 of §88 on page 273.

Calculate:

4.

$$\int_0^{2\pi} \frac{\mathrm{d}\theta}{5 + 4\sin\theta}.$$

5.

$$\int_0^{2\pi} \frac{\cos^2 3\theta\,\mathrm{d}\theta}{5 - 4\cos 2\theta}.$$

6.

$$\int_0^{\pi} \frac{\mathrm{d}\theta}{(a + \cos\theta)^2} \qquad \text{where} \qquad a > 1.$$

7. Show that

$$\int_0^{\pi} \sin^{2n}\theta \, \mathrm{d}\theta = \frac{(2n)!}{2^{2n}(n!)^2}\pi,$$

where $n = 1, 2, 3, \ldots$.

Challenge Problems: (Just for fun)

8. Calculate

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{x^4 + ax^2 + b^2} \qquad \text{where} \qquad a > 0, b > 0, a^2 \geq 4b^2.$$