HOMEWORK 6, DUE FRIDAY FEBRUARY 23RD

1. Chapter 6, Section 8: 1, 2, 3.
Update: 6.8.2 contains a typo. The first limit should read

$$\lim_{s\to 1^+}(s-1)\prod_q\left(1-\frac{1}{q^s}\right)^{-2}=A.$$

Note the exponent is -2 and not -1.
2. Let

$$f\colon \mathbb{R}\longrightarrow\mathbb{R}$$

be the function

$$f(x)=\begin{cases}e^{-1/x^2} & \text{if } x\neq 0\\ 0 & \text{if } x=0.\end{cases}$$

(a) Show that there are polynomials $p_n(x)$ and $q_n(x)$ such that the nth derivative

$$f^{(n)}(x)=\frac{p_n(x)}{q_n(x)}e^{-1/x^2}$$

for $x\neq 0$.
(b) Show that

$$f^{(n)}(0)=0.$$

(c) Show that there is a real function with an everywhere convergent Taylor series but which is not equal to its Taylor series.