PRACTICE PROBLEMS FOR THE SECOND MIDTERM

1. Give the definition of:
   (i) a curve of genus zero.
   (ii) p-adic valuation.
   (iii) p-adic absolute value.
   (iv) p-adic integer.
   (v) p-adic number.
   (vi) Cauchy sequence.
   (vii) components of an element of $\mathbb{Z}[\sqrt{d}]$.
   (viii) conjugate of an element of $\mathbb{Z}[\sqrt{d}]$.
   (ix) norm of an element of $\mathbb{Z}[\sqrt{d}]$.
   (x) fundamental solution of Pell's equation.

2. (i) Find the first few terms of the 7-adic expansion of 3/28.
(ii) Find the first few terms of a 13-adic number such that $x^2 = -1$.

3. Show that there are curves of genus 0 which don't have parametrisations by rational functions with rational coefficients.

4. State Legendre's theorem. Do the following equations have integral solutions?
(i)
$$x^2 + 2y^2 + 3z^2 = 0.$$
(ii)
$$5x^2 + 7y^2 - 3z^2 = 0.$$
(iii)
$$-5x^2 + 7y^2 - 3z^2 = 0.$$

5. Show that there is an integer $|k| \leq 1 + 2\sqrt{d}$ such that
$$x^2 - dy^2 = k$$
has infinitely many solutions.

6. Show that
$$x^2 - dy^2 = 1$$
has a non-trivial solution.

7. Show that a p-adic number α belongs to $\mathbb{Q}$ if and only if its coefficients are eventually periodic.

8. Assuming that $x^2 - dy^2 = -1$ has a solution, find an expression for the general solution.