FIRST HOMEWORK, DUE WEDNESDAY JANUARY 18TH

1. Let F be a field. Show that $F[[x]]$ is a Euclidean domain.
2. Let F be a field. Prove that to give an R-module M over the polynomial ring $F[x]$ is exactly the same as to give a vector space V, together with a linear map $\phi\colon V \longrightarrow V$.
3. Let M and N be any two R-modules. Denote by $\operatorname{Hom}_R(M, N)$ the set of all R-linear maps from M to N. Show that this set is naturally an R-module.
4. Let M be an R-module and let X be a subset of M. The annihilator I of X is the subset of all elements r of R such that $rm = 0$, for all elements m of X. Show that I is an ideal of R. Prove also that the annihilator of X is equal to the annihilator of the submodule generated by X.