HOMEWORK 4, DUE TUESDAY MAY 2ND

1. If x is a non-negative real number then prove that
$$(1+x)^n \geq 1 + nx,$$
for all non-negative integers n.

2. List the elements of

(a)
$$\{\, A \in \mathcal{P}\{\, 1, 2, 3, 4 \,\} \mid |A| \text{ is even} \,\}.$$

(b)
$$\{\, A \in \mathcal{P}\{\, 1, 2, 3, 4 \,\} \mid |A| \text{ is odd} \,\}.$$

3. True or false?

(a)
$$|\{\, \{\, 1 \,\}, \{\, x \in \mathbb{R} \mid x > 0, (x^2 - 1)^2 = 0 \,\} \,\}| = 1.$$

(b)
$$\{\, \emptyset \,\} \subset \{\, \{\, \emptyset \,\}, 2 \,\}.$$

(c)
$$|\{\, 1, \mathbb{R}, \{\, x \in \mathbb{R} \mid x^2 \geq 0 \,\} \,\}| = 2.$$

4. Let A, B and C be three sets. Prove that

(a) $A \triangle \emptyset = A$,

(b) $A \triangle A = \emptyset$,

(c) if $A \triangle B = A \triangle C$ then $B = C$.

5. If
$$A \subset \{\, 1, 2, 3, \ldots, n \,\}$$
then prove that

$\quad |A| \text{ is even} \qquad \text{if and only if} \qquad |A \triangle \{\, 1 \,\}| \text{ is odd}.$

Challenge problems/Just for fun:

6. You are on a desert island, with a scale, and a rock weighing 40 pounds.

If the rock breaks into four pieces of integral weight and you are able to weigh anything whose weight is an integer from 1 to 40, what are the weights of the pieces?

If a rock of weight w pounds breaks into n pieces and you can weigh everything up to weight w pounds, what is the largest w can be and what are the weights of the pieces?

7. Jan and Maria have fallen in love (via the internet) and Jan wishes to mail her a ring. Unfortunately they live in the country of Kleptopia

where anything sent in the mail is stolen unless it is enclosed in a padlocked box. Jan and Maria have plenty of padlocks, but none to which the other has a key. How can Jan get the key safely to Maria?