HOMEWORK 2, DUE TUESDAY APRIL 18TH

1. (4.1), (4.2), (4.3).
2. Prove that if n is an integer then $n(n+1)$ is even.
3. Let d, m, n, r and s be integers. Show that if d divides m and d divides n then d divides $rm + sn$.
4. True or false? If a and b are integers and 6 divides ab then either 6 divides a or 6 divides b.
5. Let n be an integer. Show that if there is a divisor d of n with $1 < d < n$ then there is a divisor d' of n with $1 < d' \le \sqrt{n}$.
6. Let x and y be positive real numbers. Show that

$$\sqrt{xy} \ge \frac{2}{\frac{1}{x} + \frac{1}{y}}.$$

Challenge problems/Just for fun:
7. You are in a team of twelve people. The team will stand in a straight line facing forward (so that you can see the backs of the people in front of you), and someone will tape a piece of paper to your back. The paper will be either red or blue. Each person in the line must then say either red or blue. Each person in the line can only speak once, but they can speak in any order. Everyone else in the line can hear the colour you announce. The team wins if at least eleven people say the colour that is on their back.
Describe a winning strategy.
8. You have 12 coins; one coin has a different weight to the other 11. How you can tell which coin is different and whether it is heavier or lighter using a scale (which can just compare the weights of two collections of coins) with only three weighings?
9. Let G be a graph. Show that the sum of the degrees of all of the vertices is even (*Hint: compare the sum to the number m of edges of G*).