HOMEWORK 1, DUE TUESDAY APRIL 11TH

All numbers refer to Eccles.

1. (a) Prove that

$$|a| \geq a,$$

for all real numbers a.

(b) Prove that

$$|b|^2 = b^2,$$

for all real numbers b.

(c) Prove that

$$|c| + |d| \geq |c + d|,$$

for all real numbers c and d.

2. (3.2) and (3.3).

3. Let d, a_1, a_2, b_1 and b_2 be integers. Show that if

$$d \mid (a_1 - a_2) \qquad \text{and} \qquad d \mid (b_1 - b_2)$$

then

$$d \mid ((a_1 + b_1) - (a_2 + b_2)) \qquad \text{and} \qquad d \mid (a_1 b_1 - a_2 b_2).$$

Challenge problems/Just for fun:

4. You are a prisoner in a room with 2 doors and 2 guards. One door leads to freedom and behind the other is a hangman, but you don't know which door is which.

One of the guards always tells the truth and the other always lies. Once again, you don't know which guard is which.

You have to choose and open one of these doors, but you can only ask a single question of one of the guards.

What question should you ask?

5. Consider strings of letters formed from the alphabet

$$\{\, M, I, U \,\}.$$

Starting with the string MI and applying any one of the rules below in any order as many times as you like, is it possible to generate the string MU? (This problem appears in GEB):

Rules:

(1) Replace a string of the form xI with xIU.
(2) Replace a string of the form Mx with Mxx.
(3) Replace a string of the form $xIIIy$ with xUy.
(4) Replace a string of the form $xUUy$ with xy.

For example, starting with MI we can create MIU (apply the first rule with $x = M$). Then we can create $MIUIU$ (apply the second rule with $x = IU$). Or we could start with MI, apply the second rule to get MII, apply the second rule again to get $MIIII$. Now apply the third rule to get MUI.