PRACTICE PROBLEMS FOR THE SECOND MIDTERM

1. Give the definition of:
   (i) associates of an integral domain.
   (ii) an irreducible element of an integral domain.
   (iii) a prime element of an integral domain.
   (iv) unique factorisation domain.
   (v) principal ideal domain.
   (vi) Euclidean domain.
   (vii) a partial order.
   (viii) a total order.
   (ix) the ascending chain condition.
   (x) the gcd of a pair of elements of an integral domain.
   (xi) the content of a polynomial over a UFD.

2. Let R be an integral domain.
(i) Describe the factorisation algorithm.
(ii) Show that the factorisation algorithm always terminates if and only if the set of principal ideals satisfies the ACC.
3. Let R be an integral domain. Show that every element of R has at most one factorisation into primes, up to order and associates.
4. Show that the set of all ideals satisfies the ACC in a PID.
5. (i) Prove that $F[x]$ is a Euclidean domain, where F is a field.
(ii) Show that a polynomial $f(x) \in F[x]$ has a linear factor if and only if it has a zero.
(iii) Show that a polynomial of degree two or three is irreducible if and only if it does not have any zeroes.
6. (i) State Gauss' Lemma.
(ii) Show that if R is a UFD then so is $R[x]$.
(iii) Show that if R is a UFD then so is $R[x_1, x_2, \ldots, x_n]$.