PRACTICE PROBLEMS FOR THE FIRST MIDTERM

1. Give the definition of:
   (i) a ring.
   (ii) a commutative ring.
   (iii) a zero divisor.
   (iv) an integral domain.
   (v) the characteristic of a ring.
   (vi) an ideal.
   (vii) the quotient of a ring by an ideal.
   (viii) a prime ideal.
   (ix) a maximal ideal.
   (x) the field of fractions of an integral domain.

2. Let R be a ring and let I and J be two ideals.
(i) Show that the intersection $I \cap J$ is an ideal.
(ii) Is the union $I \cup J$ an ideal?

3. Let R be an integral domain and let I be an ideal. Show that R/I is a field if and only if I is a maximal ideal.

4. (i) Let R be an integral domain. If $ab = ac$, for $a \neq 0$, b, $c \in R$, then show that $b = c$.
(ii) Show that every finite integral domain is a field.

5. If
$$I_1 \subset I_2 \subset I_3 \subset \cdots \subset I_n \subset \cdots ,$$
is an ascending sequence of ideals in a ring R then the union I is an ideal.

6. Let R be a ring and let $S = M_{2,2}(R)$ be the ring of all 2×2 matrices with entries in R. If I is an ideal of S then show that there is an ideal J of R such that I consists of all 2×2 matrices with entries in J.