HOMEWORK 8, DUE TUESDAY MARCH 1ST

1. Let M be an R-module and let $r \in R$. Show that the map

$$\phi \colon M \longrightarrow M \qquad \text{given by} \qquad m \longrightarrow rm$$

is R-linear.
2. Prove that a subset N of an R-module is a submodule if and only if it is non-empty and closed under addition and scalar multiplication.
3. Let $\phi \colon M \longrightarrow N$ be an R-linear map between two R-modules. Prove that the kernel of ϕ is a submodule of M.
4. Let M be an R-module. Prove that the intersection of any set of submodules is a submodule.
5. Let M be an R-module and let X be any subset of M. Prove the existence of the submodule generated by X.
6. Let M be an R-module and let X be any set. Show how the set of all maps from X to M becomes an R-module.
7. Let M and N be any two R-modules. Denote by $\text{Hom}_R(M, N)$ the set of all R-linear maps from M to N. Show that this set is naturally an R-module.
8. Let M be an R-module and let X be a subset of M. The annihilator I of X, is the subset of all elements r of R, such that $rm = 0$, for all elements m of X. Show that I is an ideal of R. Prove also that the annihilator of X is equal to the annihilator of the submodule generated by X.