PRACTICE PROBLEMS FOR THE 2ND MIDTERM

1. Give the definition of:
   (i) a ring.
   (ii) the product of two rings.
   (iii) a ring homomorphism.
   (iv) a commutative ring.
   (v) a ring with unity.
   (vi) a unit.
   (vii) a field.
   (viii) a zero-divisor.
   (ix) the cancellation laws.
   (x) an integral domain.
   (xi) the characteristic of a ring.
   (xii) the Euler phi-function.
   (xiii) field of fractions.

2. Classify the following group according to the fundamental theorem of finitely generated abelian groups

$$\frac{\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_4}{\langle 3, 0, 0 \rangle}.$$

3. Give an example of a group G having no elements of finite order > 1 but having a quotient group G/H, all of whose elements are of finite order.

4. Determine whether the given operations are defined on the set and whether they give a ring. If we don't get a ring, explain why. If we do get a ring, state whether the ring is commutative, whether it has unity and whether it is a field.
(i) $2\mathbb{Z} \times \mathbb{Z}$ with the usual addition and multiplication.
(ii) The set of all pure imaginary complex numbers

$$\{\, ri \,|\, r \in \mathbb{R} \,\}$$

with the usual addition and multiplication.

5. Describe all units in the ring $\mathbb{Z} \times \mathbb{Q} \times \mathbb{R}$.

6. Show that the set of all units U in a ring R is a group under multiplication.

7. Let R be a commutative ring with $1 \neq 0$. Show that R is an integral domain if and only if R satisfies the cancellation laws.

8. Show that $2^{11,213} - 1$ is not divisible by 11.