

MODEL ANSWERS TO THE THIRD HOMEWORK

§14: 2. The order of $\mathbb{Z}_4 \times \mathbb{Z}_{12}$ is 48. The order of $\langle 2 \rangle \times \langle 2 \rangle$ is $2 \times 6 = 12$, since 2 has order 2 in \mathbb{Z}_4 and 2 has order 6 in \mathbb{Z}_{12} . The order of the quotient group is $48/12 = 4$.

6. The order of $\mathbb{Z}_{12} \times \mathbb{Z}_{18}$ is $12 \cdot 18$. The order of $(4, 3)$ is 6, since the order of 4 in \mathbb{Z}_{12} is 3 and the order of 3 in \mathbb{Z}_{18} is 6, and the lcm of 3 and 6 is 6. Hence the order of $\langle (4, 3) \rangle$ is 6. The order of the quotient group is $(12 \cdot 18)/6 = 12 \cdot 3 = 36$.

11. 3. We have

$$(2, 1) + \langle (1, 1) \rangle + (2, 1) + \langle (1, 1) \rangle = (4, 2) + \langle (1, 1) \rangle$$

and

$$(4, 2) + \langle (1, 1) \rangle + (2, 1) + \langle (1, 1) \rangle = (6, 3) + \langle (1, 1) \rangle = (0, 0) + \langle (1, 1) \rangle.$$

Thus

$$3((2, 1) + \langle (1, 1) \rangle) = (0, 0) + \langle (1, 1) \rangle.$$

On the other hand $(2, 1) \notin \langle (1, 1) \rangle$ so that the order of $(2, 1) + \langle (1, 1) \rangle$ is 3.

23. T: (a), (b), (c), (d), (e), (g), (i),

F: (f), (h), (j).