HOMEWORK #5, DUE WEDNESDAY MARCH 12TH

1. Prove that if X and Y are topological manifolds of dimension m and n then $X \times Y$ is a topological manifold of dimension $m + n$.
2. If E is a compact set in a region U, prove that there is a constant M, depending only on E and U, such that every positive harmonic function u satisfies

$$u(z_2) \leq Mu(z_1),$$

for any two points z_1 and $z_2 \in E$.
3. Show that the functions $|x|$, $|z|^\alpha, (\alpha \geq 0)$, $\log(1 + |z|^2)$ are subharmonic.
4. If $v(z)$ is upper semicontinuous on the open set U, show that it has a maximum on every compact subset $E \subset U$.
5. Formulate and prove a theorem to the effect that a uniform limit of subharmonic functions is subharmonic.