

HOMWORK #3, DUE WEDNESDAY FEBRUARY 5TH

1. If $f(z)$ is holomorphic in the whole plane and real on the real axis, purely imaginary on the imaginary axis, show that $f(z)$ is odd.
2. Evaluate the following integrals, using the method of residues.

(i)

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx.$$

(ii)

$$\int_0^{\infty} \frac{x^{1/3} dx}{1 + x^2}.$$

(iii)

$$\int_0^{\infty} \log(1 + x^2) \frac{dx}{x^{1+\alpha}} \quad (0 < \alpha < 2).$$

(Hint: Try integration by parts.)

(iv)

$$\int_{|z|=\rho} \frac{|dz|}{|z - a|^2}, \quad |a| \neq \rho.$$

(Hint: Use $z\bar{z} = \rho^2$ to convert the integral to a line integral of a rational function.)