HOMEWORK #1, DUE WEDNESDAY JANUARY 21ST

1. Show that
$$\prod_{n=2}^{\infty}\left(1-\frac{1}{n^2}\right)=\frac{1}{2}.$$
2. What is the genus of $\cos\sqrt{z}$?
3. Show that the bounded regions determined by a closed curve are simply connected whilst the unbounded region is not.
4. Show that analytic branches of $\log(z)$, z^α and z^z can be defined in any simply connected domain which does not contain the origin.
5. Prove the formula of Gauss:
$$(2\pi)^{\frac{n-1}{2}}\Gamma(z)=n^{(z-1/2)}\Gamma\left(\frac{z}{n}\right)\Gamma\left(\frac{z+1}{n}\right)\cdots\Gamma\left(\frac{z+n-1}{n}\right).$$
6. What are the residues of $\Gamma(z)$ at the poles $z=-n$?