

FOURTH HOMEWORK, DUE WEDNESDAY MAY 7TH

This homework works through the explicit calculation of a Seshadri constant. We will need some of the standard theory of Jacobians and abelian surfaces.

1. Let C be a smooth curve of genus 2 over \mathbb{C} . Let

$$C_2 = \frac{C \times C}{\langle \sigma \rangle},$$

by the symmetric product of C with itself, where σ is the natural involution switching the factors.

- (i) Show that C_2 is a smooth surface.
- (ii) Show that points of C_2 correspond to divisors of degree two.
- (iii) If $\pi: C_2 \rightarrow \text{Pic}^2 C$ is the natural map (that is, the Abel-Jacobi map) which sends a divisor $p + q$ of degree two to the corresponding line bundle $\mathcal{O}_C(p + q)$ of degree two, then show that π collapses a copy of \mathbb{P}^1 to a point (use the fact that $\text{Pic}^2 C$ is isomorphic to the Jacobian, an abelian surface).
- (iv) If C is generic, then the Picard number of C_2 is two, and the class of the image of a fibre x and the diagonal δ span the Néron-Severi group. Identify $\overline{\text{NE}}(C_2)$ in this case.
- (v) What is the Seshadri constant of a principally polarised abelian surface of Picard number one (you may use the fact that any such arises as the Jacobian of a curve of genus 2).