HOMEWORK #6, DUE WEDNESDAY NOVEMBER 19TH

1. Prove that an entire holomorphic function f which satisfies the inequality

$$|f(z)| < |z|^n$$

for some n and all sufficiently large $|z|$ is a polynomial.
2. Show that an entire holomorphic function with a pole at ∞ (in other words, not an essential singularity) is a polynomial.
3. Show that e^z, $\cos z$ and $\sin z$ have essential singularities at ∞.
4. Show that a function which is meromorphic on the extended complex plane is a rational function.
5. Determine explicitly the largest disk U about the origin such that the restriction of the entire holomorphic function $f(z) = z^2 + z$ to U is injective.