HOMEWORK #5, DUE WEDNESDAY NOVEMBER 5TH

1. Find a conformal map that carries the region common to $|z| < 1$ and $|z-1| < 1$ to the interior of the unit disc, in such a way that both symmetries are preserved.

2. Compute

(i)

$$\int_\gamma x\,dz$$

where γ is the directed line segment from 0 to $1+i$.

(ii)

$$\int_\gamma x\,dz$$

where γ is the circle of radius r, centre the origin, in two ways. First parametrically and secondly using the identity $x = \frac{1}{2}(z+\bar{z}) = \frac{1}{2}(z+\frac{r^2}{z})$, valid on the circle.

(iii)

$$\int_\gamma \frac{dz}{z^2-1}$$

where γ is the circle of radius 2, centre the origin.

(iv)

$$\int_\gamma \frac{e^z}{z^2-1}\,dz$$

where γ is the circle of radius 2, centre the origin.

(v)

$$\int_\gamma e^z z^{-n}\,dz$$

where γ is the circle of radius 1, centre the origin.

(vi)

$$\int_\gamma z^n(1-z)^{-m}\,dz$$

where γ is the circle of radius 2, centre the origin.