HOMEWORK #2, DUE WEDNESDAY OCTOBER 15TH

1. Consider projection (stereographic projection) from the point $(0, 0, 1)$ between the points of the sphere S, given by $x^2 + y^2 + z^2 = 1 \subset \mathbb{R}^3$ (excluding the point $(0, 0, 1)$) and the plane $\mathbb{C} = \mathbb{R}^2$.
(i) Show that z and z' are the projection of antipodal points if and only if $z\bar{z}' = -1$.
(ii) A cube has its vertices on the sphere S and its edges are parallel to the coordinate axes. Find the projection of the vertices of the cube.
2. Show that the group of Möbius transformations is exactly thrice transitive, that is, show there is a unique Möbius transformation that carries any three distinct points (of the extended complex plane) to any other three distinct points.
3. If $f(z) = \sum a_n z^n$, what does $\sum_n n^3 a_n z^n$ represent?
4. Give an example of an unbounded sequence which does not converge to infinity.
5. Suppose that the sequence $z_1, z_2, \dots$ converges to infinity. What does this say about the sequences $\operatorname{Re} z_n$, $\operatorname{Im} z_n$, $|z_n|$ and $\arg z_n$?