

**Math 20E Homework Assignment 3**  
**Due 11:00pm Thursday, February 16, 2023**

1. Evaluate the path integral  $\int_{\mathbf{c}} f(x, y, z) ds$  with  $f(x, y, z) = x + y + z$  and  $\mathbf{c}(t) = (\sin(t), \cos(t), t)$  for  $t \in [0, 2\pi]$ .
2. Evaluate  $\int_{\mathbf{c}} f ds$ , where  $f(x, y, z) = z$  and  $\mathbf{c}(t) = (t \cos(t), t \sin(t), t)$  for  $0 \leq t \leq t_0$ .
3. Find the average  $z$  coordinate on the path  $\mathbf{c}(t) = (t \cos(t), t \sin(t), t)$  for  $0 \leq t \leq t_0$ .
4. Evaluate  $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$ , where  $\mathbf{F}(x, y) = (-xy, x^2)$  and  $\mathbf{c}$  is the path along the unit circle  $x^2 + y^2 = 1$  beginning at  $(1, 0)$  and ending at  $(0, 1)$ .
5. Evaluate the line integral  $\int_{\mathbf{c}} yz dx + xz dy + xy dz$ , where  $\mathbf{c}$  consists of the straight-line segments joining  $(1, 0, 0)$  to  $(0, 1, 0)$  to  $(0, 0, 1)$ .
6. Evaluate the line integral  $\int_C (y^2 + 2xz) dx + (2xy + z^2) dy + (2yz + x^2) dz$ , where  $C$  is an oriented simple curve from  $(1, 1, 1)$  to  $(0, 2, 3)$ .
7. Let  $S$  be the surface determined by the equation  $x^3 + 3xy + z^2 = 2$ , with  $z \geq 0$ .
  - (a) Find a parametrization  $\Phi : D \subseteq \mathbb{R}^2 \rightarrow S \subseteq \mathbb{R}^3$ .
  - (b) Find an equation for the tangent plane to  $S$  at the point  $(1, 1/3, 0)$ .
8. Find the area of the portion of the unit sphere that is inside the mouth of the cone  $z \geq \sqrt{x^2 + y^2}$ .
9. The cylinder  $x^2 + y^2 = x$  divides the unit sphere  $S$  into two regions  $S_1$  and  $S_2$ , where  $S_1$  is outside the cylinder and  $S_2$  is inside the cylinder.  
Find the ratio  $A(S_1)/A(S_2)$  of the areas of  $S_1$  and  $S_2$ .
10. Find the area of the surface  $S$  defined by  $x + y + z = 1$ , with  $x^2 + 3y^2 \leq 1$ .