

**Instructions**

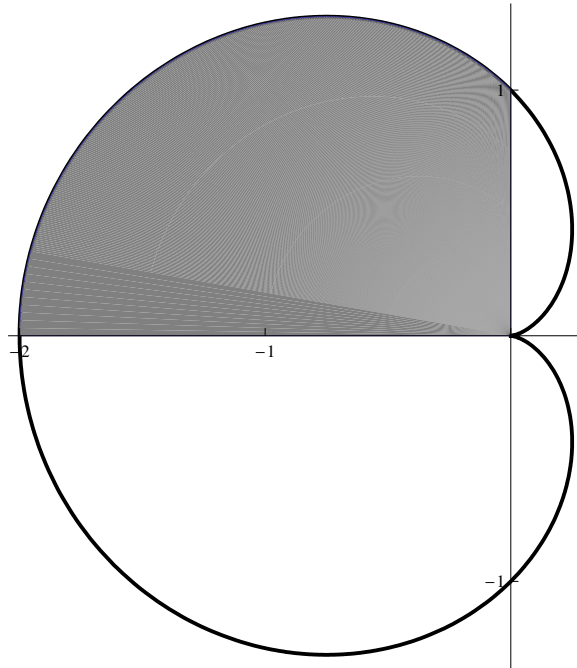
1. Write your Name and PID in the spaces provided above.
  2. Make sure your Name is on every page.
  3. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
  4. Put away ANY devices that can be used for communication or can access the Internet.
  5. You may use one handwritten page of notes, but no books or other assistance during this exam.
  6. Read each question carefully and answer each question completely.
  7. Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.
  8. Show all of your work. No credit will be given for unsupported answers, even if correct.
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(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Use complex exponentials to evaluate the indefinite integral  $\int e^{2ix} \cos^2(5x) dx$ .

Leave your answer in complex exponential form; there should be no trigonometric functions in your final answer.

- (6 points) 2. Compute the area of the shaded portion of the cardioid  $r = 1 - \cos(\theta)$  shown in the figure below.



- (6 points) 3. Determine the convergence or divergence of each of the following improper integrals. In order to earn credit, you must provide a correct justification for each answer.

(a)  $\int_1^{\infty} \frac{1}{x^4 + \sqrt{x}} dx$

(b)  $\int_0^1 \frac{1}{x^4 + \sqrt{x}} dx$

(c)  $\int_0^{\infty} \frac{1}{x^4 + \sqrt{x}} dx$

4. Let  $f(x) = \frac{x^2 + 2x + 3}{(x - 2)(x^2 + 1)}$ .

(4 points) (a) Find the partial fraction decomposition of  $f(x)$ .

(2 points) (b) Compute  $\int f(x) dx$ .

5. Evaluate the following integral.

(4 points) (a)  $\int \frac{1}{x^2\sqrt{x^2+4}} dx$

5. Evaluate the following integral.

(4 points) (b)  $\int \tan^{-1}(2x) dx$

(4 points) 6. Let  $\{a_n\}$  be the sequence with  $a_n = n \sin\left(\frac{2\pi}{n}\right)$ . Determine whether the sequence  $\{a_n\}$  converges or diverges. If it converges, determine its limit.

(6 points) 7. Recall that the Taylor series centered at  $x = 0$  for  $\sin(x)$  is given by

$$\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

(a) Write the power series centered at  $x = 0$  that represents  $\sin(x^2)$ .

(b) Evaluate the integral  $\int_0^x \sin(t^2) dt$  using the power series found in part (a).

8. Consider the power series  $\sum_{n=1}^{\infty} 2^{-\ln(n)} x^n$ .

(6 points)

(a) Determine the radius of convergence of the power series.

(b) (Extra Credit: 6 points) Determine if the power series converges or diverges at each endpoint of the interval of convergence.